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Radford 双积 monoidal BiHom-Hopf 代数

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摘　要:首先给出 smash 积 monoidal BiHom-代数$(B\# H,\alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$和 smash 余积 monoidal BiHom-余代数$(B\times H,\alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$,进而得到$(B\# H,\alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$和$(B\times H,\alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$构成 monoidal BiHom-双代数的充分必要条件.

关键词:Radford 双积;Monoidal BiHom-Hopf 代数;Hopf 代数

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构造 Hopf 代数是 Hopf 代数理论中的一个重要研究内容. 1985 年文献[1]通过 smash 积和 smash 余积 构造了一类 Hopf 代数，称之为 Radford 双积，等价于构造了一个 Yetter-Drinfeld 范畴中的 Hopf 代数，使得该结构在有限维有点 Hopf 代数的分类中起到了关键作用. 对 Radford 双积的其他研究可见文献[2－6].

Hom 型代数起源于文献[7]对 Witt 代数和 Virasoro 代数的 q-形变的研究时提出的 Hom-Lie 代数，这是近年来 Hopf 代数理论的一个研究热点,受到许多研究者的关注,见文献[6－14]. 其中文献[13]从范畴角度研究 Hom 代数,从而引入了 monoidal Hom-Hopf 代数的概念. 2015 年,文献[14]又得到了这一结构的 monoidal BiHom 型，并且给出了一类 BiHom smash 积结构. 2014 年,在文献[6]给出了一种 Radford 双积的 monoidal Hom-型. 很自然地要问 Radford 双积的 monoidal Bi-Hom 型是什么? 本文主要给出 Radford 双积的一类 monoidal BiHom 型,推广了文献[6]中的主要结果.

文中所有代数系统都是定义在域 K 上的. 若 C 是余代数，则用 Sweedler 记号(文献[15])表示余乘:$\Delta(c)=c_1\otimes c_2,\forall c\in C$. 记左 H-余模范畴为${}^H\mathrm{Mod}$，即$(M,\rho)\in {}^H\mathrm{Mod},\rho(x)=x_{(-1)}\otimes x_{(0)}\in H\otimes M,\forall x\in M$. 给定一个 K-空间 M，记 M 上恒等态射为 Id_M.

1　基本定义

以下给出文献[14]中的定义的 monoidal 版本.

定义 1　Monoidal BiHom-代数是一个五元组 $(A,\mu,1_A,\alpha,\beta)$ (缩写为 (A,α,β)),其中 A 是一个 K-线性空间,$\mu:A\otimes A\to A$ 是一个 K-线性映射记 $\mu(a\otimes a')=aa'$, $1_A\in A$,α,β 是 A 的自同构,对任意的 a,a', $a''\in A$ 且满足下面条件:

(A1) $\alpha\circ\beta=\beta\circ\alpha$,

(A2) $\alpha(aa')=\alpha(a)\alpha(a')$; $\alpha(1_A)=1_A$,

(A3) $\beta(aa')=\beta(a)\beta(a')$; $\beta(1_A)=1_A$,

(A4) $\alpha(a)(a'a'')=(aa')\beta(a'')$,

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(A5) $a1_A = \alpha(a)$；$1_A a = \beta(a)$.

注 1　当 $\alpha = \beta$ 时，Monoidal BiHom- 代数是 Monoidal Hom- 代数.

定义 2　Monoidal BiHom- 余代数是一个五元组 $(C,\Delta,\varepsilon,\psi,\omega)$（缩写为 (C,ψ,ω)），其中 C 是一个 K- 线性空间，Δ: $C \to C \otimes C$，ε: $C \to K$ 是一个 K- 线性映射记 $\Delta(c) = c_1 \otimes c_2$，$\psi,\omega$ 是 C 的自同构，对任意的 $c \in C$ 且满足下面条件：

(C1) $\psi \circ \omega = \omega \circ \psi$,

(C2) $\psi(c)_1 \otimes \psi(c)_2 = \psi(c_1) \otimes \psi(c_2)$；$\varepsilon \circ \psi = \varepsilon$,

(C3) $\omega(c)_1 \otimes \omega(c)_2 = \omega(c_1) \otimes \omega(c_2)$；$\varepsilon \circ \omega = \varepsilon$,

(C4) $\psi^{-1}(c_1) \otimes c_{21} \otimes c_{22} = c_{11} \otimes c_{12} \otimes \omega^{-1}(c_2)$,

(C5) $\varepsilon(c_1)c_2 = \omega^{-1}(c)$；$c_1\varepsilon(c_2) = \psi^{-1}(c)$.

注 2　当 $\psi = \omega$ 时，Monoidal BiHom- 余代数是 Monoidal Hom- 余代数.

定义 3　Monoidal BiHom- 双代数是一个 7 元组 $(H,\mu,1_H,\Delta,\varepsilon,\alpha,\beta)$（缩写为 (H,α,β)），其中 $(H,\mu,1_H,\alpha,\beta)$ 是一个 monoidal BiHom- 代数，$(H,\Delta,\varepsilon,\alpha,\beta)$ 是一个 monoidal BiHom- 余代数，并且使得满足

$$\Delta(hh') = \Delta(h)\Delta(h')；\Delta(1_H) = 1_H \otimes 1_H，\varepsilon(hh') = \varepsilon(h)\varepsilon(h')；\varepsilon(1_H) = 1.$$

此外，如果存在线性映射 S: $H \to H$ 使得

$$S(h_1)h_2 = h_1S(h_2) = \varepsilon(h)1_H，S \circ \alpha = \alpha \circ S，和 S \circ \beta = \beta \circ S，$$

则称 $(H,\mu,1_H,\Delta,\varepsilon,\alpha,\beta,S)$ (abbr. (H,α,β,S)) 是 Monoidal BiHom-Hopf 代数.

定义 4　设 (A,α_A,β_A) 是 monoidal BiHom- 代数. 一个左 A- 模是一个四元组 $(M,\triangleright,\alpha_M,\beta_M)$，其中 M 是一个线性空间，$\triangleright$: $A \otimes M \to M$ 是线性映射，且 α_M,β_M 是 M 的自同构，对任意的 $a,a' \in A$ 和 $m \in M$ 使得

(M1) $\alpha_M \circ \beta_M = \beta_M \circ \alpha_M$,

(M2) $\alpha_M(a \triangleright m) = \alpha_A(a) \triangleright \alpha_M(m)$,

(M3) $\beta_M(a \triangleright m) = \beta_A(a) \triangleright \beta_M(m)$,

(M4) $\alpha_A(a) \triangleright (a' \triangleright m) = (aa') \triangleright \beta_M(m)$；$1_A \triangleright m = \beta_M(m)$.

注 3　显然 (A,μ,α_A,β_A) 是 左 (A,α_A,β_A)- 模.

定义 5　设 (C,ψ_C,ω_C) 是一个 monoidal BiHom- 余代数. 一个左 C- 余模是三元组 (M,ψ_M,ω_M)，其中 M 是一个线性空间，ρ: $M \to C \otimes M$（记 $\rho(m) = m_{(-1)} \otimes m_{(0)}$，$m \in M$）是线性映射，且 ψ_M,ω_M 是 M 的自同构，对任意的 $m \in M$ 使得

(CM1) $\psi_M \circ \omega_M = \omega_M \circ \psi_M$,

(CM2) $\psi_M(m)_{(-1)} \otimes \psi_M(m)_{(0)} = \psi_C(m_{(-1)}) \otimes \psi_M(m_{(0)})$,

(CM3) $\omega_M(m)_{(-1)} \otimes \omega_M(m)_{(0)} = \omega_C(m_{(-1)}) \otimes \omega_M(m_{(0)})$,

(CM4) $\psi_C^{-1}(m_{(-1)}) \otimes m_{(0)(-1)} \otimes m_{(0)(0)} = m_{(-1)1} \otimes m_{(-1)2} \otimes \omega_M^{-1}(m_{(0)})$；

$\varepsilon_C(m_{(-1)})m_{(0)} = \omega_M^{-1}(m)$.

注 4　显然 $(C,\Delta,\psi_C,\omega_C)$ 是左 -(C,ψ_C,ω_C) 余模.

定义 6　设 (H,α_H,β_H) 是 monoidal BiHom- 双代数. 一个 monoidal BiHom- 代数 (B,α_B,β_B) 称为左 -H 模 monoidal BiHom- 代数，如果 (B,α_B,β_B) 是一个有 $\triangleright$ 作用的左 -H 模，并且对任意的 $b,b' \in B$；$h \in H$ 满足下面条件：

$$h \triangleright (bb') = (h_1 \triangleright b)(h_2 \triangleright b')，h \triangleright 1_B = \varepsilon(h)1_B.$$

定义 7　设 (H,α_H,β_H) 是 monoidal BiHom- 双代数. 一个 monoidal BiHom- 余代数 (B,α_B,β_B) 称为左 -H 余模 monoidal BiHom- 余代数，如果 (B,α_B,β_B) 是一个左 -H 余模（余作用为 $\rho(b) = b_{(-1)} \otimes b_{(0)}$），并且对任意的 $b \in B$ 满足下面条件：

$$b_{(-1)} \otimes \Delta_B(b_{(0)}) = b_{1(-1)}b_{2(-1)} \otimes b_{1(0)} \otimes b_{2(0)}，b_{(-1)}\varepsilon(b_{(0)}) = \varepsilon(b)1_H.$$

定义 8　设 (H,α_H,β_H) 是 monoidal BiHom- 双代数. 一个 monoidal BiHom- 代数 (B,α_B,β_B) 称为左 -H 余模 monoidal BiHom- 代数，如果 (B,α_B,β_B) 是一个左 -H 余模且余作用为 ρ，并且对任意的 $a,b \in B$ 满足下面条件：

$$\rho(ab) = a_{(-1)}b_{(-1)} \otimes a_{(0)}b_{(0)}, \rho(1_B) = 1_H \otimes 1_B.$$

定义 9 设(H,α_H,β_H)是 monoidal BiHom- 双代数. 一个 monoidal BiHom- 余代数 (B,α_B,β_B) 称为左 -H 模monoidal BiHom- 余代数,如果(B,α_B,β_B)是一个有 $\triangleright$ 作用的左 -H 模,并且对任意的 $b \in B;h \in H$ 满足下面条件:

$$\Delta(h \triangleright b) = h_1 \triangleright b_1 \otimes h_2 \triangleright b_2, \ \varepsilon_B(h \triangleright b) = \varepsilon_H(h)\varepsilon_B(b).$$

2 主要结果

命题 1 设(H,α_H,β_H)是 monoidal BiHom- 双代数,(B,α_B,β_B)是左 H-BiHom- 模代数,则$(B \# H,\alpha_B \otimes \alpha_H,\beta_B \otimes \beta_H)$ $(= B \otimes H$ 作为向量空间$)$是一个 monoidal BiHom- 代数,单位元为 $1_B \otimes 1_H$ 且乘法为

$$(a \# h)(b \# g) = a(\beta_H^{-1}(\alpha_H(h_1)) \triangleright \beta_B^{-1}(b)) \# \beta_H(h_2)g,$$

其中 $a, b \in B, h, g \in H$. 这种情况下,称$(B \# H,\alpha_B \otimes \alpha_H,\beta_B \otimes \beta_H)$为 smash 积 monoidal BiHom- 代数.

证明 显然可得到$(\alpha_B \otimes \alpha_H) \circ (\beta_B \otimes \beta_H) = (\beta_B \otimes \beta_H) \circ (\alpha_B \otimes \alpha_H)$,

$$(\alpha_B \otimes \alpha_H)(1_B \otimes 1_H) = 1_B \otimes 1_H \text{ 和} (\beta_B \otimes \beta_H)(1_B \otimes 1_H) = 1_B \otimes 1_H.$$

$$(a \otimes h)(1_B \otimes 1_H) = a(\beta_H^{-1}(\alpha_H(h_1)) \triangleright \beta_B^{-1}(1_B)) \otimes \beta_H(h_2)1_H = a(\beta_H^{-1}(\alpha_H(h_1)) \triangleright 1_B) \otimes \alpha_H(\beta_H(h_2)) = a1_B\varepsilon(h_1) \otimes \alpha_H(\beta_H(h_2)) = (\alpha_B \otimes \alpha_H)(a \otimes h).$$

类似可得

$$(1_B \otimes 1_H)(a \otimes h) = (\beta_B \otimes \beta_H)(a \otimes h).$$

下证:$(\alpha_B \otimes \alpha_H)((a \otimes h)(b \otimes g)) = ((\alpha_B \otimes \alpha_H)(a \otimes h))((\alpha_B \otimes \alpha_H)(b \otimes g))$. 事实上,

$$(\alpha_B \otimes \alpha_H)((a \otimes h)(b \otimes g)) = (\alpha_B \otimes \alpha_H)(a(\beta_H^{-1}(\alpha_H(h_1)) \triangleright \beta_B^{-1}(b)) \otimes \beta_H(h_2)g) = \alpha_B(a(\beta_H^{-1}(\alpha_H(h_1)) \triangleright \beta_B^{-1}(b))) \otimes \alpha_H(\beta_H(h_2)g) = \alpha_B(a)\alpha_B(\beta_H^{-1}(\alpha_H(h_1)) \triangleright \beta_B^{-1}(b)) \otimes \alpha_H(\beta_H(h_2))\alpha_H(g) = \alpha_B(a)(\beta_H^{-1}(\alpha_H^2(h_1)) \triangleright \alpha_B(\beta_B^{-1}(b))) \otimes \alpha_H(\beta_H(h_2))\alpha_H(g) = ((\alpha_B \otimes \alpha_H)(a \otimes h))((\alpha_B \otimes \alpha_H)(b \otimes g)).$$

类似可证:

$$(\beta_B \otimes \beta_H)((a \otimes h)(b \otimes g)) = ((\beta_B \otimes \beta_H)(a \otimes h))((\beta_B \otimes \beta_H)(b \otimes g)).$$

下证:$((\alpha_B \otimes \alpha_H)(a \otimes h))((b \otimes k)(c \otimes g)) = ((a \otimes h)(b \otimes k))((\beta_B \otimes \beta_H)(c \otimes g))$. 事实上,

$$((\alpha_\beta \otimes \alpha_H)(a \otimes h))((b \otimes k)(c \otimes g)) = (\alpha_B(a) \otimes \alpha_H(h))(b(\beta_H^{-1}(\alpha_H(k_1)) \triangleright \beta_B^{-1}(c)) \otimes \beta_H(k_2)g) = \alpha_B(a)(\beta_H^{-1}(\alpha_H(\alpha_H(h_1))) \triangleright \beta_B^{-1}(b(\beta_H^{-1}(\alpha_H(k_1)) \triangleright \beta_B^{-1}(c)))) \otimes \beta_H(\alpha_H(h_2))(\beta_H(k_2)g) = \alpha_B(a)(\beta_H^{-1}(\alpha_H^2(h_1)) \triangleright (\beta_B^{-1}(b)(\beta_H^{-2}(\alpha_H(k_1)) \triangleright \beta_B^{-2}(c)))) \otimes \beta_H(\alpha_H(h_2))(\beta_H(k_2)g) = \alpha_B(a)((\beta_H^{-1}(\alpha_H^2(h_1))_1 \triangleright \beta_B^{-1}(b))(\beta_H^{-1}(\alpha_H^2(h_1))_2 \triangleright (\beta_H^{-2}(\alpha_H(k_1)) \triangleright \beta_B^{-2}(c)))) \otimes \beta_H(\alpha_H(h_2))(\beta_H(k_2)g) = \alpha_\beta(a)((\beta_H^{-1}(\alpha_H^2(h_{11})) \triangleright \beta_B^{-1}(b))(\beta_H^{-1}(\alpha_H(h_{12})) \cdot \beta_H^{-2}(\alpha_H(k_1)) \triangleright \beta_B^{-1}(c))) \otimes \beta_H(\alpha_H(h_2))(\beta_H(k_2)g).$$

和

$$((a \otimes h)(b \otimes k))((\beta_B \otimes \beta_H)(c \otimes g)) = (a(\beta_H^{-1}(\alpha_H(h_1)) \triangleright \beta_B^{-1}(b)) \otimes \beta_H(h_2)k)(\beta_B(c) \otimes \beta_H(g)) = (a(\beta_H^{-1}(\alpha_H(h_1)) \triangleright \beta_B^{-1}(b)))(\beta_H^{-1}(\alpha_H((\beta_H(h_2)k)_1)) \triangleright c) \otimes \beta_H((\beta_H(h_2)k)_2)\beta_H(g) = (\alpha_H(\beta_H^{-1}(\alpha_H(h_1)) \triangleright \beta_B^{-1}(b)))(\beta_H^{-1}(\alpha_H(\beta_H(h_{21})k_1)) \triangleright c) \otimes \beta_H(\beta_H(h_{22})k_2)\beta_H(g) = \alpha_B(a)((\beta_H^{-1}(\alpha_H(h_1)) \triangleright \beta_B^{-1}(b))(\beta_H^{-1}(\alpha_H(h_{21}))\beta_H^{-2}(\alpha_H(k_1)) \triangleright \beta_B^{-1}(c))) \otimes (\beta_H^2(h_{22})\beta_H(k_2))\beta_H(g) = \alpha_B(a)((\beta_H^{-1}(\alpha_H^2(h_{11})) \triangleright \beta_B^{-1}(b))(\beta_H^{-1}(\alpha_H(h_{12}))\beta_H^{-2}(\alpha_H(k_1)) \triangleright \beta_B^{-1}(c))) \otimes (\beta_H(h_2)\beta_H(k_2))\beta_H(g) = \alpha_B(a)((\beta_H^{-1}(\alpha_H^2(h_{11})) \triangleright \beta_B^{-1}(b))(\beta_H^{-1}(\alpha_H(h_{12}))\beta_H^{-2}(\alpha_H(k_1)) \triangleright \beta_B^{-1}(c))) \otimes \beta_H(\alpha_H(h_2))(\beta_H(k_2)g).$$

证毕.

命题 2 设(H,α_H,β_H)是 monoidal BiHom- 双代数,(B,α_B,β_B)是左 H-BiHom- 余模余代数,则$(B \times H, \alpha_B \otimes \alpha_H,\beta_B \otimes \beta_H)(= B \otimes H$ 作为向量空间$)$是一个 monoidal BiHom- 余代数,余单位元为 $\varepsilon_B \otimes \varepsilon_H$ 且余乘为 $\Delta(b \otimes h) = b_1 \otimes \alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1) \otimes \beta_B(b_{2(0)}) \otimes h_2$,其中 $b \in B,h \in H$. 这种情况下,称$(B \times H, \alpha_B \otimes \alpha_H,\beta_B \otimes \beta_H)$为 smash 余积 monoidal BiHom- 余代数.

证明　显然可得到$(\alpha_B\otimes\alpha_H)\circ(\beta_B\otimes\beta_H)=(\beta_B\otimes\beta_H)\circ(\alpha_B\otimes\alpha_H)$，$(\varepsilon_B\otimes\varepsilon_H)\circ(\alpha_B\otimes\alpha_H)=\varepsilon_B\otimes\varepsilon_H$ 和$(\varepsilon_B\otimes\varepsilon_H)\circ(\beta_B\otimes\beta_H)=\varepsilon_B\otimes\varepsilon_H$.

下证：$(Id_{B\otimes H}\otimes\varepsilon_{B\otimes H})\Delta(b\otimes h)=(\alpha_B^{-1}\otimes\alpha_H^{-1})(b\otimes h)$.

$$(Id_{B\otimes H}\otimes\varepsilon_{B\otimes H})\Delta(b\otimes h)=(Id_{B\otimes H}\otimes\varepsilon_{B\otimes H})(b_1\otimes\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1)\otimes\beta_B(b_{2(0)})\otimes h_2)=b_1\otimes\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1)\varepsilon(b_{2(0)})\varepsilon(h_2)=(\alpha_B^{-1}\otimes\alpha_H^{-1})(b\otimes h).$$

类似可得：$(\varepsilon_{B\otimes H}\otimes Id_{B\otimes H})\Delta(b\otimes h)=(\beta_B^{-1}\otimes\beta_H^{-1})(b\otimes h)$.

下证：

$((\beta_B\otimes\beta_H)\otimes(\beta_B\otimes\beta_H))\Delta(b\otimes h)=\Delta((\beta_B\otimes\beta_H)(b\otimes h))$，

$$((\beta_B\otimes\beta_H)\otimes(\beta_B\otimes\beta_H))\Delta(b\otimes h)=((\beta_B\otimes\beta_H)\otimes(\beta_B\otimes\beta_H))(b_1\otimes\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1)\otimes\beta_B(b_{2(0)})\otimes h_2)=\beta_B(b_1)\otimes\alpha_H^{-1}(\beta_H^2(b_{2(-1)}))h_1\otimes\beta_B^2(b_{2(0)})\otimes\beta_H(h_2)=\Delta((\beta_B\otimes\beta_H)(b\otimes h)).$$

类似可证：$((\alpha_B\otimes\alpha_H)\otimes(\alpha_B\otimes\alpha_H))\Delta(b\otimes h)=\Delta((\alpha_B\otimes\alpha_H)(b\otimes h))$.

下证：

$$(\Delta\otimes(\beta_B\otimes\beta_H)^{-1})\Delta(b\otimes h)=((\alpha_B\otimes\alpha_H)^{-1}\otimes\Delta)\Delta(b\otimes h).$$

$$\begin{aligned}&(\Delta\otimes(\beta_B\otimes\beta_H)^{-1})\Delta(b\otimes h)=(\Delta\otimes\beta_B^{-1}\otimes\beta_H^{-1})(b_1\otimes\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1)\otimes\beta_B(b_{2(0)})\otimes h_2)=\\&\Delta(b_1\otimes\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1))\otimes b_{2(0)}\otimes\beta_H^{-1}(h_2)=b_{11}\otimes\alpha_H^{-1}(\beta_H(b_{12(-1)}))\beta_H^{-1}((\alpha_H^{-1}(\beta_H(b_{2(-1)}))\cdot\\&\beta_H^{-1}(h_1))_1)\otimes\beta_B(b_{12(0)})\otimes(\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1))_2\otimes b_{2(0)}\otimes\beta_H^{-1}(h_2)=b_{11}\otimes\\&\alpha_H^{-1}(\beta_H(b_{12(-1)}))\beta_H^{-1}(\alpha_H^{-1}(\beta_H(b_{2(-1)1}))\beta_H^{-1}(h_{11}))\otimes\beta_B(b_{12(0)})\otimes\alpha_H^{-1}(\beta_H(b_{2(-1)2}))\beta_H^{-1}(h_{12})\otimes\\&b_{2(0)}\otimes\beta_H^{-1}(h_2)=\alpha_B^{-1}(b_1)\otimes\alpha_H^{-1}(\beta_H(b_{21(-1)}))\beta_H^{-1}(\alpha_H^{-1}(\beta_H(\beta_B(b_{22})_{(-1)1}))\beta_H^{-1}(h_{11}))\otimes\\&\beta_B(b_{21(0)})\otimes\alpha_H^{-1}(\beta_H(\beta_B(b_{22})_{(-1)2}))\beta_H^{-1}(h_{12})\otimes\beta_B(b_{22})_{(0)}\otimes\beta_H^{-1}(h_2)=\\&\alpha_B^{-1}(b_1)\otimes\alpha_H^{-1}(\beta_H(b_{21(-1)}))(\alpha_H^{-1}(\beta_H(b_{22(-1)1}))\beta_H^{-2}(h_{11}))\otimes\beta_B(b_{21(0)})\otimes\\&\alpha_H^{-1}(\beta_H^2(b_{22(-1)2}))\beta_H^{-1}(h_{12})\otimes\beta_B(b_{22(0)})\otimes\beta_H^{-1}(h_2)=\alpha_B^{-1}(b_1)\otimes\\&(\alpha_H^{-2}(\beta_H(b_{21(-1)}))\alpha_H^{-1}(\beta_H(b_{22(-1)1})))\beta_H^{-1}(h_{11})\otimes\beta_B(b_{21(0)})\otimes\\&\alpha_H^{-1}(\beta_H^2(b_{22(-1)2}))\beta_H^{-1}(h_{12})\otimes\beta_B(b_{22(0)})\otimes\beta_H^{-1}(h_2).\end{aligned}$$

又有

$$\begin{aligned}&((\alpha_B\otimes\alpha_H)^{-1}\otimes\Delta)\Delta(b\otimes h)=(\alpha_B^{-1}\otimes\alpha_H^{-1}\otimes\Delta)(b_1\otimes\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1)\otimes\\&\beta_B(b_{2(0)})\otimes h_2)=\alpha_B^{-1}(b_1)\otimes\alpha_H^{-2}(\beta_H(b_{2(-1)}))\alpha_H^{-1}(\beta_H^{-1}(h_1))\otimes\Delta(\beta_B(b_{2(0)})\otimes h_2)=\\&\alpha_B^{-1}(b_1)\otimes\alpha_H^{-2}(\beta_H(b_{2(-1)}))\alpha_H^{-1}(\beta_H^{-1}(h_1))\otimes\beta_B(b_{2(0)1})\otimes\alpha_H^{-1}(\beta_H(\beta_B(b_{2(0)2(-1)}))\beta_H^{-1}(h_{21})\otimes\\&\beta_B(\beta_B(b_{2(0)2})_{(0)})\otimes h_{22}=\alpha_B^{-1}(b_1)\otimes\alpha_H^{-2}(\beta_H(b_{2(-1)}))\alpha_H^{-1}(\beta_H^{-1}(h_1))\otimes\beta_B(b_{2(0)1})\otimes\\&\alpha_H^{-1}(\beta_H^2(b_{2(0)2(-1)}))\beta_H^{-1}(h_{21})\otimes\beta_B^2(b_{2(0)2(0)})\otimes h_{22}=\alpha_B^{-1}(b_1)\otimes\alpha_H^{-2}(\beta_H(b_{21(-1)}b_{22(-1)}))\cdot\\&\alpha_H^{-1}(\beta_H^{-1}(h_1))\otimes\beta_B(b_{21(0)})\otimes\alpha_H^{-1}(\beta_H^2(b_{22(0)(-1)}))\beta_H^{-1}(h_{21})\otimes\beta_B^2(b_{22(0)(0)})\otimes h_{22}=\\&\alpha_B^{-1}(b_1)\otimes\alpha_H^{-2}(\beta_H(b_{21(-1)}b_{22(-1)}))\beta_H^{-1}(h_{11})\otimes\beta_B(b_{21(0)})\otimes\alpha_H^{-1}(\beta_H^2(b_{22(0)(-1)}))\beta_H^{-1}(h_{12})\otimes\\&\beta_B^2(b_{22(0)(0)})\otimes\beta_H^{-1}(h_2)=\alpha_B^{-1}(b_1)\otimes(\alpha_H^{-2}(\beta_H(b_{21(-1)}))\alpha_H^{-1}(\beta_H(b_{22(-1)1})))\beta_H^{-1}(h_{11})\otimes\\&\beta_B(b_{21(0)})\otimes\alpha_H^{-1}(\beta_H^2(b_{22(-1)2}))\beta_H^{-1}(h_{12})\otimes\beta_B(b_{22(0)})\otimes\beta_H^{-1}(h_2).\end{aligned}$$

证毕.

注 5　当 $\alpha_H=\beta_H$，$\alpha_B=\beta_B$时，此命题是文献[6]中的命题 3.2.

定理 1　设(H,α_H,β_H)是 monoidal BiHom-双代数，$(B\# H,\alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$是 smash 积 monoidal BiHom-代数，$(B\times H,\alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$是 smash 余积 monoidal BiHom-余代数，则下面条件等价：

(i) $(B^{\#}_{\times}H,m_{B\# H},1_B\otimes 1_H,\Delta_{B\times H},\varepsilon_B\otimes\varepsilon_H,\alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$是 monoidal BiHom-双代数.

(ii) 下面条件成立：

(c_1) ε_B 是代数同态，$\Delta_B(1_B)=1_B\otimes 1_B$，

(c_2) (B,α_B,β_B)是左 H-BiHom-模余代数，

(c_3) (B,α_B,β_B)是左 H-BiHom-余模代数，

(c_4) $\Delta_B(ab)=a_1(a_{2(-1)}\triangleright\beta_B^{-1}(b_1))\otimes\beta_B(a_{2(0)})b_2$，

$(c_5)\alpha_H^{-1}((\alpha_H(h_1)\triangleright b)_{(-1)})h_2\otimes(\alpha_H(h_1)\triangleright b)_{(0)}=h_1b_{(-1)}\otimes h_2\triangleright b_{(0)}$,

其中 $a,b\in B$, $h\in H$. 这种情况下称$(B\#_\times H,\alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$为 Radford 双积 monoidal BiHom- 双代数.

证明 ((i)⇒(ii)) 因为 $\varepsilon_{B\times H}$ 是代数同态，所以 $\varepsilon_B(1_B)=\varepsilon_{B\times H}(1_B\otimes 1_H)=1$ 且

$$\varepsilon_B(a(\beta_H^{-1}(h)\triangleright\beta_B^{-1}(b)))\varepsilon_H(g)=\varepsilon_B(a)\varepsilon_B(b)\varepsilon_H(h)\varepsilon_H(g).\tag{1}$$

在(1)式中令 $h=g=1_H$,则 $\varepsilon_B(ab)=\varepsilon_B(a)\varepsilon_B(b)$.

在(1)式中令 $a=1_B$, $g=1_H$,则可得到 $\varepsilon_B(h\triangleright b)=\varepsilon_H(h)\varepsilon_B(b)$.

因为 $\Delta_{B\times H}$ 是代数同态,所以

$$1_{B1}\otimes\beta_H(1_{B2(-1)})\otimes\beta_B(1_{B2(0)})\otimes 1_H=1_B\otimes 1_H\otimes 1_B\otimes 1_H.\tag{2}$$

应用 $\varepsilon_B\otimes Id\otimes Id\otimes\varepsilon_H$ 作用于(2)式的两边,可得到 $1_{B(-1)}\otimes 1_{B(0)}=1_H\otimes 1_B$.

应用 $Id\otimes\varepsilon_H\otimes Id\otimes\varepsilon_H$ 作用于(2)式的两边,可得到 $\Delta(1_B)=1_B\otimes 1_B$.

由 $\Delta((a\otimes h)(b\otimes g))=\Delta(a\otimes h)\Delta(b\otimes g)$ 得,

$$\begin{aligned}&(a(\beta_H^{-1}(\alpha_H(h_1))\triangleright\beta_B^{-1}(b)))_1\otimes\alpha_H^{-1}(\beta_H((a(\beta_H^{-1}(\alpha_H(h_1))\triangleright\beta_B^{-1}(b)))_{2(-1)}))\beta_H^{-1}((\beta_H(h_2)g)_1)\otimes\\&\beta_B((a(\beta_H^{-1}(\alpha_H(h_1))\triangleright\beta_B^{-1}(b)))_{2(0)})\otimes(\beta_H(h_2)g)_2=a_1(a_{2(-1)1}\alpha_H(\beta_H^{-2}(h_{11}))\triangleright\beta_B^{-1}(b_1))\otimes\\&(\alpha_H^{-1}(\beta_H{}^2(a_{2(-1)2}))h_{12})(\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(g_1))\otimes\beta_B(a_{2(0)})(\beta_H^{-1}(\alpha_H(h_{21}))\triangleright b_{2(0)})\otimes\beta_H(h_{22})g_2.\end{aligned}\tag{3}$$

在(3)式中令 $h=g=1_H$ 可得:

$$\begin{aligned}&(ab)_1\otimes\beta_H((ab)_{2(-1)})\otimes\beta_B((ab)_{2(0)})\otimes 1_H=a_1(\alpha_H(a_{2(-1)1})\triangleright\beta_B^{-1}(b_1))\otimes\\&\beta_H^2(a_{2(-1)2})\beta_H(b_{2(-1)})\otimes\beta_B(a_{2(0)})\beta_B(b_{2(0)})\otimes 1_H.\end{aligned}\tag{4}$$

用 $\varepsilon_B\otimes Id\otimes Id\otimes\varepsilon_H$ 作用于(4)式两端可得:

$$(ab)_{(-1)}\otimes(ab)_{(0)}=a_{(-1)}b_{(-1)}\otimes a_{(0)}b_{(0)}.$$

再用 $Id\otimes\varepsilon_H\otimes Id\otimes\varepsilon_H$ 作用于(4)式两端可得,可得 (c_4).

在(3)式中令 $a=1_B$, $g=1_H$, 有:

$$\begin{aligned}&(\alpha_H(h_1)\triangleright b)_1\otimes\alpha_H^{-1}(\beta_H((\alpha_H(h_1)\triangleright b)_{2(-1)}))\beta_H^{-1}((\alpha_H(\beta_H(h_2)))_1)\otimes\\&\beta_B((\alpha_H(h_1)\triangleright b)_{2(0)})\otimes(\alpha_H(\beta_H(h_2)))_2=(\alpha_H(h_{11})\triangleright b_1)\otimes\\&\beta_H(h_{12})\beta_H(b_{2(-1)})\otimes(\alpha_H(h_{21})\triangleright\beta_B(b_{2(0)}))\otimes\alpha_H(\beta_H(h_{22})).\end{aligned}\tag{5}$$

用 $Id\otimes\varepsilon_H\otimes Id\otimes\varepsilon_H$ 作用于(5)式两端,可得 $\Delta_B(h\triangleright b)=h_1\triangleright b_1\otimes h_2\triangleright b_2$.

用 $\varepsilon_B\otimes Id\otimes Id\otimes\varepsilon_H$ 作用于(5)式两端,可得(c_5).

因此条件$(c_1)-(c_5)$成立.

((ii)⇒(i)) 易证 $\varepsilon_{B\times H}$ 是代数同态和 $\Delta_{B\times H}(1_B\otimes 1_H)=1_B\otimes 1_H\otimes 1_B\otimes 1_H$.

下证:$\Delta_{B\times H}((a\otimes h)(b\otimes g))=\Delta_{B\times H}(a\otimes h)\Delta_{B\times H}(b\otimes g)$.

$$\begin{aligned}&\Delta_{B\times H}((a\otimes h)(b\otimes g))=\Delta_{B\times H}(a(\beta_H^{-1}(\alpha_H(h_1))\triangleright\beta_B^{-1}(b))\otimes\beta_H(h_2)g=(a(\beta_H^{-1}(\alpha_H(h_1))\triangleright\beta_B^{-1}(b)))_1\otimes\\&\alpha_H^{-1}(\beta_H((\alpha_H(\beta_H^{-1}(\alpha(h_1))\triangleright\beta_B^{-1}(b)))_{2(-1)}))\beta_H^{-1}((\beta_H(h_2)g)_1)\otimes\beta_B((a(\beta_H^{-1}(\alpha_H(h_1))\triangleright\beta_B^{-1}(b)))_{2(0)})\otimes\\&(\beta_H(h_2)g)_2=a_1(a_{2(-1)}\triangleright\beta_B^{-1}((\beta_H^{-1}(\alpha_H(h_1))\triangleright\beta_B^{-1}(b))_1))\otimes\alpha_H^{-1}(\beta_H((\beta_B(a_{2(0)})(\beta_H^{-1}(\alpha_H(h_1))\triangleright\\&\beta_B^{-1}(b))_2)_{(-1)}))\beta_H^{-1}((\beta_H(h_2)g)_1)\otimes\beta_B((\beta_B(a_{2(0)})(\beta_H^{-1}(\alpha_H(h_1))\triangleright\beta_B^{-1}(b))_2)_{(0)})\otimes(\beta_H(h_2)g)_2=\\&a_1(a_{2(-1)}\triangleright\beta_B^{-1}((\beta_H^{-1}(\alpha_H(h_{11}))\triangleright\beta_B^{-1}(b_1))))\otimes\alpha_H^{-1}(\beta_H((\beta_B(a_{2(0)})(\beta_H^{-1}(\alpha_H(h_{12}))\triangleright\\&\beta_B^{-1}(b_2)))_{(-1)}))(h_{21}\beta_H^{-1}(g_1))\otimes\beta_B((\beta_B(a_{2(0)})(\beta_H^{-1}(\alpha_H(h_{12}))\triangleright\beta_B^{-1}(b_2)))_{(0)})\otimes\\&\beta_H(h_{22})g_2=a_1(a_{2(-1)}\triangleright\beta_B^{-1}((\beta_H^{-1}(\alpha_H(h_{11}))\triangleright\beta_B^{-1}(b_1))))\otimes\alpha_H^{-1}(\beta_H(\beta_B(a_{2(0)})_{(-1)}\cdot\\&(\beta_H^{-1}(\alpha_H(h_{12}))\triangleright\beta_B^{-1}(b_2))_{(-1)}))(h_{21}\beta_H^{-1}(g_1))\otimes\beta_B(\beta_B(a_{2(0)})_{(0)}(\beta_H^{-1}(\alpha_H(h_{12}))\triangleright\\&\beta_B^{-1}(b_2))_{(0)})\otimes\beta_H(h_{22})g_2=a_1(a_{2(-1)}\triangleright\beta_B^{-1}((\beta_H^{-1}(\alpha_H(h_{11}))\triangleright\beta_B^{-1}(b_1))))\otimes\\&\alpha_H^{-1}(\beta_H(\beta_H(a_{2(0)(-1)})(\beta_H^{-1}(\alpha_H(h_{12}))\triangleright\beta_B^{-1}(b_2))_{(-1)}))(h_{21}\beta_H^{-1}(g_1))\otimes\\&\beta_B(\beta_B(a_{2(0)(0)})(\beta_H^{-1}(\alpha_H(h_{12}))\triangleright\beta_B^{-1}(b_2))_{(0)})\otimes\beta_H(h_{22})g_2=\\&a_1(a_{2(-1)}\triangleright\beta_B^{-1}((\beta_H^{-1}(\alpha_H(h_{11}))\triangleright\beta_B^{-1}(b_1))))\otimes(\alpha_H^{-1}(\beta_H^2(a_{2(0)(-1)}))\\&\alpha_H^{-1}(\beta_H((\beta_H^{-1}(\alpha_H^2(h_{121}))\triangleright\beta_B^{-1}(b_2))_{(-1)})))(\beta_H(h_{122})\beta_H^{-1}(g_1))\otimes\\&\beta_B(\beta_B(a_{2(0)(0)})(\beta_H^{-1}(\alpha_H(h_{121}))\triangleright\beta_B^{-1}(b_2))_{(0)})\otimes h_2g_2=a_1(a_{2(-1)}\triangleright\end{aligned}$$

$$\beta_B^{-1}((\beta_H^{-1}(\alpha_H(h_{11}))\triangleright\beta_B^{-1}(b_1))))\otimes(\alpha_H^{-1}(\beta_H^2(a_{2(0)(-1)}))(\alpha_H^{-2}(\beta_H((\beta_H^{-1}\cdot$$
$$(\alpha_H^2(h_{121}))\triangleright\beta_B^{-1}(b_2))_{(-1)}))h_{122}))g_1\otimes\beta_B^2(a_{2(0)(0)})\beta_B((\beta_H^{-1}(\alpha_H^2(h_{121}))\triangleright$$
$$\beta_B^{-1}(b_2))_{(0)})\otimes h_2g_2=a_1(a_{2(-1)}\triangleright(\beta_H^{-2}(\alpha_H(h_{11}))\triangleright\beta_B^{-2}(b_1)))\otimes$$
$$(\alpha_H^{-1}(\beta_H^2(a_{2(0)(-1)}))(\alpha_H^{-2}(\beta_B(\beta_H^{-1}(\alpha_H^2(h_{121}))\triangleright\beta_B^{-1}(b_2))_{(-1)})h_{122}))g_1\otimes$$
$$\beta_B^2(a_{2(0)(0)})\beta_B(\beta_H^{-1}(\alpha_H^2(h_{121}))\triangleright\beta_B^{-1}(b_2))_{(0)}\otimes h_2g_2=a_1(a_{2(-1)}\triangleright$$
$$(\beta_H^{-2}(\alpha_H(h_{11}))\triangleright\beta_B^{-2}(b_1)))\otimes(\alpha_H^{-1}(\beta_H^2(a_{2(0)(-1)}))(\alpha_H^{-1}(\alpha_H^{-1}((\alpha_H^2(h_{121})\triangleright$$
$$b_2)_{(-1)})\alpha_H(h_{122}))))g_1\otimes\beta_B^2(a_{2(0)(0)})(\alpha_H^2(h_{121})\triangleright b_2)_{(0)}\otimes h_2g_2=$$
$$a_1(a_{2(-1)}\triangleright(\beta_H^{-2}(\alpha_H(h_{11}))\triangleright\beta_B^{-2}(b_1)))\otimes(\alpha_H^{-1}(\beta_H^2(a_{2(0)(-1)}))(\alpha_H^{-1}(\alpha_H(h_{121})b_{2(-1)})))g_1\otimes$$
$$\beta_B^2(a_{2(0)(0)})(\alpha_H(h_{122})\triangleright b_{2(0)})\otimes h_2g_2=a_1(\alpha_H(a_{2(-1)1})\triangleright(\beta_H^{-2}(\alpha_H(h_{11}))\triangleright$$
$$\beta_B^{-2}(b_1)))\otimes(\alpha_H^{-1}(\beta_H^2(a_{2(-1)2}))(h_{121}\alpha_H^{-1}(b_{2(-1)})))g_1\otimes\beta_B(a_{2(0)})(\alpha_H(h_{122})\triangleright$$
$$b_{2(0)})\otimes h_2g_2=a_1((a_{2(-1)1}\beta_H^{-2}(\alpha_H(h_{11})))\triangleright\beta_B^{-1}(b_1))\otimes(\alpha_H^{-1}(\beta_H^2(a_{2(-1)2}))\cdot$$
$$(h_{121}\alpha_H^{-1}(b_{2(-1)})))g_1\otimes\beta_B(a_{2(0)})(\alpha_H(h_{122})\triangleright b_{2(0)})\otimes h_2g_2=$$
$$a_1((a_{2(-1)1}\beta_H^{-2}(\alpha_H(h_{11})))\triangleright\beta_B^{-1}(b_1))\otimes(\alpha_H^{-1}(\beta_H^2(a_{2(-1)2}))(\alpha_H^{-1}(h_{12})\alpha_H^{-1}(b_{2(-1)})))g_1\otimes$$
$$\beta_B(a_{2(0)})(\alpha_H(\beta_H^{-1}(h_{21}))\triangleright b_{2(0)})\otimes\beta_H(h_{22})g_2=a_1((a_{2(-1)1}\beta_H^{-2}(\alpha_H(h_{11})))\triangleright$$
$$\beta_B^{-1}(b_1))\otimes(\alpha_H^{-1}(\beta_H^2(a_{2(-1)2}))h_{12})\cdot(\beta_H(\alpha_H^{-1}(b_{2(-1)}))\beta_H^{-1}(g_1))\otimes$$
$$\beta_B(a_{2(0)})(\alpha_H(\beta_H^{-1}(h_{21}))\triangleright b_{2(0)})\otimes\beta_H(h_{22})g_2=\Delta_{B\times H}(a\otimes h)\Delta_{B\times H}(b\otimes g).$$

所以$(B\#_{\times}H,\ \alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$是 monoidal BiHom- 双代数.

证毕.

注 6　当 $\alpha_H=\beta_H$，$\alpha_H=\beta_B$时，此定理是文献[6]中的定理 3.5.

命题 3　设$(B\#_{\times}H,\ \alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$是 Radford 双积 monoidal BiHom- 双代数. H 是带有对极S_H的 monoidal BiHom-Hopf 代数，在 $Hom(B,B)$ 中 $S_B:B\to B$ 是 Id 的卷积逆，且满足 $S_B\circ\beta_B=\beta_B\circ S_B$ 和 $S_B\circ\alpha_B=\alpha_B\circ S_B$，则$(B\#_{\times}H,\ \alpha_B\otimes\alpha_H,\beta_B\otimes\beta_H)$是 monoidal BiHom-Hopf 代数，对极 S 定义为$S(b\otimes h)=(1_B\otimes S_H(\beta_H^{-1}(b_{(-1)})\alpha_H^{-2}(h)))(S_B(b_{(0)})\otimes 1_H)$.

证明　对任意的$b\in B$和$h\in H$，有

$$(Id*S)(b\otimes h)=(b_1\otimes\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1))S(\beta_B(b_{2(0)})\otimes h_2)=$$
$$(b_1\otimes\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1))((1_B\otimes S_H(\beta_H^{-1}(\beta_B(b_{2(0)})_{(-1)})\alpha_H^{-2}(h_2)))\cdot$$
$$(S_B(\beta_B(b_{2(0)})_{(0)})\otimes 1_H))=(b_1\otimes\alpha_H^{-1}(\beta_H(b_{2(-1)}))\beta_H^{-1}(h_1))((1_B\otimes S_H(b_{2(0)(-1)}\alpha_H^{-2}(h_2)))\cdot$$
$$(S_B(\beta_B(b_{2(0)(0)}))\otimes 1_H))=((\alpha_B^{-1}(b_1)\otimes\alpha_H^{-2}(\beta_H(b_{2(-1)}))\alpha_H^{-1}(\beta_H^{-1}(h_1)))(1_B\otimes$$
$$S_H(b_{2(0)(-1)}\alpha_H^{-2}(h_2))))(S_B(\beta_B^2(b_{2(0)(0)}))\otimes 1_H)=(b_1\otimes(\beta_H(\alpha_H^{-2}(b_{2(-1)}))\alpha_H^{-1}(\beta_H^{-1}(h_1)))\cdot$$
$$(S_H(b_{2(0)(-1)}\alpha_H^{-2}(h_2))))(S_B(\beta_B^2(b_{2(0)(0)}))\otimes 1_H)=(b_1\otimes(\beta_H(\alpha_H^{-2}(b_{2(-1)}))\alpha_H^{-1}(\beta_H^{-1}(h_1)))\cdot$$
$$(S_H(\alpha_H^{-2}(h_2))S_H(b_{2(0)(-1)})))(S_B(\beta_B^2(b_{2(0)(0)}))\otimes 1_H)=(b_1\otimes(\beta_H(\alpha_H^{-2}(b_{2(-1)}))(\alpha_H^{-2}(\beta_H^{-1}(h_1))\cdot$$
$$\beta_H^{-1}(S_H(\alpha_H^{-2}(h_2)))))S_H(\beta_H(b_{2(0)(-1)})))(S_B(\beta_B^2(b_{2(0)(0)}))\otimes 1_H)=(b_1\otimes\beta_H(b_{2(-1)1})S_H\cdot$$
$$(\beta_H(b_{2(-1)2})))(S_B(\beta_B(b_{2(0)}))\otimes 1_H)\varepsilon(h)=(b_1\otimes 1_H)(S_B(b_2)\otimes 1_H)\varepsilon(h)=$$
$$(b_1S_B(b_2)\otimes 1_H)\varepsilon(h)=(1_B\otimes 1_H)\varepsilon(b\otimes h).$$

相似的可证明$(S*Id)(b\otimes h)=(1_B\otimes 1_H)\varepsilon(b\otimes h)$. 证毕.

注 7　当 $\alpha_H=\beta_H,\alpha_B=\beta_B$时，此命题是文献[6]中的命题 3.6.

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Radford Biproduct Monoidal BiHom-Hopf Algebra

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Abstract: Firstly, we introduce the notions of a smash product monoidal BiHom-algebra $(B\# H, \alpha_B\otimes\alpha_H, \beta_B\otimes\beta_H)$ and a smash coproduct monoidal BiHom-coalgebra$(B\times H, \alpha_B\otimes\alpha_H, \beta_B\otimes\beta_H)$. Furthermore, we get sufficient and necessary conditions for $(B\# H, \alpha_B\otimes\alpha_H, \beta_B\otimes\beta_H)$ and $(B\times H, \alpha_B\otimes\alpha_H, \beta_B\otimes\beta_H)$ to be a monoidal BiHom-bialgebra.

Keywords: Radford biproduct; Monoidal BiHom-Hopf algebra; Hopf algebra

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The Cauchy Problem for the Higher-Order Dispersive Equation

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Abstract: In this paper, we consider the Cauchy problem for the higher-order dispersive equation $u_t+\partial_x^{2j+1}u=\partial_x^{j+1}(u^2)+\partial_x^{j-1}(u_x^2), j\geq 2, j\in\mathbf{N}, x, t\in\mathbf{R}$. By using the modified Fourier restriction norm method and Strichartz estimate and the modified Bourgain space, we prove that the problem is locally well-posed in modified Sobolev space $H^{(s,\frac{1}{2j})}$ with $s>-\frac{j}{2}+\frac{3}{4}$. By using the iteration technique, we also prove that the flow map is not C^2 at the origin if we assume that the problem is well-posed in $H^{(s,w)}$ with $0<w<\frac{1}{2j}$ for any $s\in\mathbf{R}$.

Keywords: Cauchy problem; local well-posedness; modified Sobolev spaces