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一类非线性随机网络系统的均方指数稳定控制

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摘　要:研究了一类非线性随机网络控制系统的均方指数稳定控制问题.通过在网络诱导时延的时变区间插入分点,把网络诱导时延转化为满足区间 Bernoulli 分布的随机变量,并根据随机变量在不同区间上的取值,利用T-S 模糊方法建立了网络控制系统新模型.把线性矩阵不等式方法应用到新模型的处理中,得到了时延依赖的指数稳定条件,给出了模糊控制的设计方法,并对一类具体的网络系统进行了数值计算和模拟仿真.

关键词:网络系统;指数稳定;随机系统;时延

中图分类号:TP273　　**文献标志码:**A

通过网络形成的闭环系统称为网络控制系统.信息通过网络在传感器,控制器,执行器等元件之间传输.由于网络的引入,使得系统的连接变得方便、容易,系统性能相对比较稳定,系统维护成本也比较低.正是由于网络系统的众多优点,近年来,网络控制系统受到广泛的关注并成为控制界研究的热点之一[1－3].

然而，由于网络本身的特点,使得网络控制系统中不可避免地出现网络诱导时延.网络诱导时延常常产生于信息在各物理元件之间的传输过程中.众所周知,网络诱导时延常常使系统性能变差甚至不稳定.近年来,许多学者致力于对网络控制系统的分析与控制进行研究,并一致认为网络诱导时延的发生常常是随机的甚至是不确定的[4－5].网络的引入不仅仅会产生网络诱导时延,同时会常常出现信息包的丢失,时延和丢包的存在会严重影响网络系统的稳定性.文献[6]研究了离散时间网络控制系统(NCSS)的均方稳定问题.根据一些耦合代数 Riccati 方程的唯一正定解,给出了一个充分必要的稳定条件.文献[7]研究了一类无线网络系统的控制问题.在无线通信网络中,在同时考虑时间延迟和数据包丢失的影响下,建立了一个具有时变延迟的切换系统模型并提出了一个保证闭环系统稳定和保证系统性能水平的充分条件.Alcaina 等[8]提出了一种新的网络控制系统的延迟独立控制结构,将基于预测因子和双速率控制技术的分组控制策略相结合.该方法可以减少网络负载和连接设备的使用,同时保持令人满意的控制性能.Li 等[9]研究了一类离散时间网络控制系统在反馈信道和前向信道中存在网络诱导时滞和数据包丢失的问题.采用预测网络控制方法,提出了一种新的有限时间状态反馈和输出反馈稳定控制器,并对延时和数据包丢失进行了主动补偿.上述结果都是关于线性网络系统的研究成果.对非线性网络系统的研究是具有挑战意义的课题.目前对非线性网络系统的研究得到了一些结果[10－12].文献[13]针对具有状态延迟和丢包的网络控制系统,提出了一类具有系统延迟和丢包的非线性网络控制系统的状态反馈控制.通过动态输出反馈来实现这类系统的稳定.得到了保证状态变量和状态估计误差收敛到原点的充分条件.文献[14]把随机混合系统的理论方法应用到对网络控制系统的建模和分析中.提出了两种不同的数学模型和理论分析工具.Yoneyama 等[15]研究了由 T-S 模糊系统描述的非线性网络系统的镇定问题,构造的 Lyapunov 函数和相应的广义控制器使得网络控制系统渐近稳定的保守性更低.Borgers 等[16]研究了大规模网络化控制系统的输入-状态稳定性.将大规模网络控制系统建模为混合子系统的互联,构造了整个系统的输入-状态稳定 Lyapunov 函数,并保证了所有子系统都是输入-状态稳定.

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本文在前人研究的基础上,旨在研究一类具有区间分布时延的网络控制系统的均方镇定问题.通过利用T-S模糊控制方法,结合随机网络时延的分布特征,把网络系统建模为随机微分方程.利用线性矩阵不等式方法探索系统均方指数稳定条件,从而设计系统的模糊控制器.

1 问题描述

考虑下面非线性时延控制系统

规则 $i(i-1,2,\cdots,r)$:

IF $z_1(t)$ is M_1^i and $z_2(t)$ is M_2^i, $\cdots$, and $z_n(t)$ is M_n^i

$$\text{THEN}\quad \begin{array}{l} \dot{x}(t)=(A_i+\Delta A_i(t))x(t)+(A_{di}+\Delta A_{di}(t))x(t-d)+(\boldsymbol{B}_i+\Delta \boldsymbol{B}_i(t))u(t) \\ x(t)=\psi(t) \quad t\in[-d,0] \end{array} \tag{1}$$

其中 $z(t)=[z_1(t),z_2(t),\cdots,z_n(t)]^{\mathrm{T}}$ 是前件变量,$x(t)\in\mathbf{R}^n$ 是系统状态.$u(t)\in\mathbf{R}^m$ 是控制输入,$M_k^i(k=1,2,\cdots,n)$ 是模糊集.$\boldsymbol{A}_i,\boldsymbol{A}_{di}\in\mathbf{R}^{n\times n}$ 是已知的常数矩阵,$\boldsymbol{B}_i\in\mathbf{R}^{n\times m}$ 是输入矩阵,r 是模糊规则数,$\psi(t)\in\mathbf{R}^n$ 是系统初始状态,d 是常数代表系统状态时延.系统的不确定性 $\Delta\boldsymbol{A}_i(t),\Delta\boldsymbol{A}_{di}(t)\in\mathbf{R}^{n\times n}$ 满足

$$[\Delta\boldsymbol{A}_i(t)\quad \Delta\boldsymbol{A}_{di}(t)\quad \Delta\boldsymbol{B}_i(t)]=\boldsymbol{D}\boldsymbol{F}(t)[\boldsymbol{E}_{i1}\quad \boldsymbol{E}_{i2}\quad \boldsymbol{E}_{i3}],$$

其中 $\boldsymbol{D},\boldsymbol{E}_{i1},\boldsymbol{E}_{i2},\boldsymbol{E}_{i3}$ 是常数矩阵.时变矩阵 $\boldsymbol{F}(t)$ 满足 $\boldsymbol{F}^{\mathrm{T}}(t)\boldsymbol{F}(t)\leq\boldsymbol{I}$. 由 T-S 模糊控制方法可得系统(1)的全局模糊模型

$$\begin{cases}\dot{x}(t)=\sum\limits_{i}^{r}\mu_i(z(t))[(\boldsymbol{A}_i+\Delta\boldsymbol{A}_i(t))x(t)+(\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di}(t))x(t-d)+(\boldsymbol{B}_i+\Delta\boldsymbol{B}_i(t)u(t))], \\ x(t)=\psi(t),t\in[-d,0],\end{cases} \tag{2}$$

其中 $\omega_i(z(t))=\prod\limits_{k=1}^{n}M_k^i(z_k(t))$,$\mu_i(z(t))=\dfrac{\omega_i(z(t))}{\sum\limits_{i=1}^{r}\omega_i(z(t))}$.

假设网络系统所有的状态是可测的.$\tau(t)$ 代表网络诱导时延,满足 $\tau(t)\in[0,\tau]$,其中 τ 为常数.设计系统(2)的模糊控制器

$$u(t)=\sum_{i}^{r}\mu_i(z(t))\boldsymbol{K}_i x(t-\tau(t)). \tag{3}$$

由控制器(3)和系统 (2)可得

$$\begin{cases}\dot{x}(t)=\sum\limits_{i}^{r}\sum\limits_{j}^{r}\mu_i(z(t))\mu_j(z(t))[(\boldsymbol{A}_i+\Delta\boldsymbol{A}_i(t))x(t)+(\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di}(t))x(t-d)+ \\ \quad(\boldsymbol{B}_i+\Delta\boldsymbol{B}_i(t))\boldsymbol{K}_j x(t-\tau(t))], \\ x(t)=\psi(t),t\in[-\bar{d},0].\end{cases} \tag{4}$$

系统状态初始条件设为 $x(t)=\psi(t)$,其中 $\psi(t)$ 是定义在$[-\bar{d},0]$上的光滑函数,$\bar{d}=\max\{\tau,d\}$.从而存在正数 $\bar{\psi}$ 满足 $\|\dot{\psi}(t)\|\leq\bar{\psi}$,$t\in[-\bar{d},0]$.

在区间 $[0,\tau]$ 内插入一个分点 τ_1 把区间分为$[0,\tau_1]$和$[\tau_1,\tau]$两个区间,假设 $\tau(t)$ 在区间$[0,\tau_1]$和$[\tau_1,\tau]$上取值的概率是可测的.

注:时延的变化范围是$[0,\tau]$,为了使分析更贴合实际,利用 τ_1 把区间分成两部分(甚至可以根据测量到时延的区间分布情况,可以多插入几个分点分成多个区间),使得每个区间上时延的波动变化不大,使得时延处理保守性更小.

定义如下集合 $\Omega_1=\{t:\tau(t)\in[0,\tau]\}$,$\Omega_2=\{t:\tau(t)\in[\tau_1,\tau]\}$,显然 $\Omega_1\cap\Omega_2=\phi$,其中 ϕ 是空集.

定义两个函数如下

$$h_1(t)=\begin{cases}\tau(t),t\in\Omega_1, \\ 0,t\notin\Omega_2,\end{cases} \quad h_2(t)=\begin{cases}\tau(t),t\in\Omega_2, \\ \tau_1,t\notin\Omega_2,\end{cases} \tag{5}$$

随机变量

$$\beta(t)=\begin{cases}1, t\in\Omega_1,\\ 0, t\in\Omega_2.\end{cases} \tag{6}$$

把函数 $h_1(t)$, $h_2(t)$ 和随机变量 $\beta(t)$ 代入系统 (3),得到

$$\begin{cases}\dot{x}(t)=\sum_{i}^{r}\sum_{j}^{r}\mu_i(z(t))\mu_j(z(t))\bar{\boldsymbol{A}}_{ij}\xi(t),\\ x(t)=\phi(t), t\in[\bar{d},0].\end{cases} \tag{7}$$

其中

$$\bar{\boldsymbol{A}}_{ij}=[\bar{\boldsymbol{A}}_i \quad \bar{\boldsymbol{A}}_{di} \quad \beta(t)\bar{\boldsymbol{B}}_iK_j \quad (1-\beta(t))\bar{\boldsymbol{B}}_iK_j], \bar{\boldsymbol{A}}_i=\boldsymbol{A}_i+\Delta\boldsymbol{A}_i(t), \bar{\boldsymbol{A}}_{di}=\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di}(t),$$

$$\bar{\mathbf{B}}_i=\boldsymbol{B}_i+\Delta\boldsymbol{B}_i(t), \xi^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-d), x\ \mathrm{T}(t-h_1(t)), x^{\mathrm{T}}(t-h_2(t))].$$

2　主要结果

定义 1[10]　对系统 (7),如果存在常数 $\alpha>0$ 和 $\gamma\geq 1$ 使得 $E\{\|x(t)\|\}\leq\gamma\sup\limits_{-\bar{d}\leq s\leq 0}E\{\|\psi(s)\|\}\mathrm{e}^{-\alpha t}$, $t\geq 0$ 成立,则系统(7) 是均方指数稳定的.

引理 1[8]　对矩阵 $\boldsymbol{X}_i$, $\boldsymbol{Y}_i$ $(1\leq i\leq r)$ 以及适当维数的矩阵 $\boldsymbol{S}>0$, 存在下述不等式

$$2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{p=1}^{r}\sum_{l=1}^{r}\mu_i\mu_j\mu_p\mu_l\boldsymbol{X}_{ij}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{Y}_{pl}\leq\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j(\boldsymbol{X}_{ij}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{Y}_{ij}+\boldsymbol{Y}_{ij}^{\mathrm{T}}\boldsymbol{S}\boldsymbol{Y}_{ij})$$

其中 $\mu_i(1\leq i\leq r)$ 表示 $\mu_i(z(t))\geq 0$, $\sum\limits_{i=1}^{r}\mu_i(z(t))=1$.

引理 2[2]　对于任意适当维数的向量 $\boldsymbol{a}$, $\boldsymbol{b}$ 和 $\boldsymbol{N}$, $\boldsymbol{X}$, $\boldsymbol{Y}$, $\boldsymbol{Z}$ 矩阵,其中 $\boldsymbol{X}$, $\boldsymbol{Z}$ 是对称的,若 $\begin{bmatrix}\boldsymbol{X} & \boldsymbol{Y}\\ \boldsymbol{Y}^{\mathrm{T}} & \boldsymbol{Z}\end{bmatrix}\geq 0$,则不等式 $-2\boldsymbol{a}^{\mathrm{T}}\boldsymbol{N}\boldsymbol{b}\leq\inf\limits_{\boldsymbol{X},\boldsymbol{Y},\boldsymbol{Z}}\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{X} & \boldsymbol{Y}-\boldsymbol{N}\\ \boldsymbol{Y}^{\mathrm{T}}-\boldsymbol{N}^{\mathrm{T}} & \boldsymbol{Z}\end{bmatrix}\begin{bmatrix}\boldsymbol{a}\\ \boldsymbol{b}\end{bmatrix}$ 成立.

引理 3[4]　线性矩阵不等式

$$\begin{bmatrix}\boldsymbol{Y}(x) & \boldsymbol{W}(x)\\ * & \boldsymbol{R}(x)\end{bmatrix}>0$$

等价于 $\boldsymbol{R}(x)>0$, $\boldsymbol{Y}(x)-\boldsymbol{W}(x)\boldsymbol{R}^{-1}(x)\boldsymbol{W}^{\mathrm{T}}(x)>0$,其中 $\boldsymbol{Y}(x)=\boldsymbol{Y}^{\mathrm{T}}(x)$, $\boldsymbol{R}(x)=\boldsymbol{R}^{\mathrm{T}}(x)$.

引理 4[13]　已知常量 $\varepsilon>0$ 和矩阵 $\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{F}$, 且满足 $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F}\leq\boldsymbol{I}$, 则下面不等式成立

$$\boldsymbol{DEF}+\boldsymbol{E}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}}\leq\varepsilon\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}}+\varepsilon^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}.$$

定理 1　对给定的常数 $\alpha>0$, $1\geq\beta\geq 0$ 和 $i,j=1,2,\cdots,q$,如果存在正定矩阵 $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{R}$, $\boldsymbol{T}\in\mathbf{R}^{n\times n}$,矩阵 $\boldsymbol{K}_j\in\mathbf{R}^{m\times n}$ 和具有适当维数的矩阵 $\boldsymbol{X}_{1ij}$, $\boldsymbol{X}_{2ij}$, $\boldsymbol{Y}_{ij}$ 使得下面矩阵不等式成立

$$\boldsymbol{\Theta}<0, \tag{8}$$

其中

$$\boldsymbol{\Theta}=\begin{bmatrix}
\begin{matrix}\boldsymbol{P}\bar{\boldsymbol{A}}_i+\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{Q}+2\alpha\boldsymbol{P}+\\ \tau_1\boldsymbol{X}_{111}+\tau\boldsymbol{X}_{211}+\boldsymbol{Y}_{11}+\\ \boldsymbol{Y}_{11}^{\mathrm{T}}+\boldsymbol{Y}_{21}+\boldsymbol{Y}_{21}^{\mathrm{T}}+\\ \tau_1\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{R}\bar{\boldsymbol{A}}_i+\tau\bar{\boldsymbol{A}}_I^{\mathrm{T}}\boldsymbol{T}\bar{\boldsymbol{A}}_i\end{matrix} &
\begin{matrix}\boldsymbol{P}\bar{\boldsymbol{A}}_{di}+\tau_1\boldsymbol{X}_{112}+\tau\boldsymbol{X}_{212}+\\ \boldsymbol{Y}_{12}^{\mathrm{T}}+\boldsymbol{Y}_{22}^{\mathrm{T}}+\tau_1\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{R}\bar{\boldsymbol{A}}_{di}+\\ \tau\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{T}\bar{\boldsymbol{A}}_{di}\end{matrix} &
\begin{matrix}\boldsymbol{P}\beta\bar{\boldsymbol{B}}_i\boldsymbol{K}_j+\tau_1\boldsymbol{X}_{113}+\tau\boldsymbol{X}_{213}-\\ \boldsymbol{Y}_{11}+\boldsymbol{Y}_{13}^{\mathrm{T}}+\boldsymbol{Y}_{23}^{\mathrm{T}}+\\ \tau_1\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{R}\beta\bar{\boldsymbol{B}}_i\boldsymbol{K}_j+\tau\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{T}\beta\bar{\boldsymbol{B}}_i\boldsymbol{K}_j\end{matrix} &
\begin{matrix}\boldsymbol{P}(1-\beta)\bar{\boldsymbol{B}}_i\boldsymbol{K}_j+\tau_1\boldsymbol{X}_{114}+\\ \tau\boldsymbol{X}_{214}+\boldsymbol{Y}_{14}^{\mathrm{T}}-\boldsymbol{Y}_{21}+\boldsymbol{Y}_{24}^{\mathrm{T}}+\\ \tau_1\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{R}(1-\beta)\bar{\boldsymbol{B}}_i\boldsymbol{K}_j+\\ \tau\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{T}(1-\beta)\bar{\boldsymbol{B}}_i\boldsymbol{K}_j\end{matrix}\\
* &
\begin{matrix}-\mathrm{e}^{-2\alpha d}\boldsymbol{Q}+\tau_1\boldsymbol{X}_{122}+\tau\boldsymbol{X}_{222}+\\ \tau_1\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}\boldsymbol{R}\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}+\tau\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}\boldsymbol{T}\boldsymbol{A}_{di}\end{matrix} &
\begin{matrix}\tau_1\boldsymbol{X}_{123}+\tau\boldsymbol{X}_{223}-\boldsymbol{Y}_{12}+\\ \tau_1\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}\boldsymbol{R}\beta\bar{\boldsymbol{B}}_i\boldsymbol{K}_j+\tau\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}\boldsymbol{T}\beta\bar{\boldsymbol{B}}_i\boldsymbol{K}_j\end{matrix} &
\begin{matrix}\tau_1\boldsymbol{X}_{124}+\tau\boldsymbol{X}_{224}-\boldsymbol{Y}_{22}+\\ \tau_1\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}\boldsymbol{R}(1-\beta)\bar{\boldsymbol{B}}_i\boldsymbol{K}_j+\\ \tau\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}\boldsymbol{T}(1-\beta)\bar{\boldsymbol{B}}_i\boldsymbol{K}_j\end{matrix}\\
* & * &
\begin{matrix}\tau_1\boldsymbol{X}_{133}+\tau\boldsymbol{X}_{233}-\boldsymbol{Y}_{13}-\boldsymbol{Y}_{13}^{\mathrm{T}}+\\ \tau_1\boldsymbol{K}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_i^{\mathrm{T}}\boldsymbol{R}\beta\bar{\boldsymbol{B}}_i\boldsymbol{K}_j+\tau\boldsymbol{K}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_i^{\mathrm{T}}\boldsymbol{T}\beta\bar{\boldsymbol{B}}_i\boldsymbol{K}_j\end{matrix} &
\tau_1\boldsymbol{X}_{134}+\tau\boldsymbol{X}_{234}-\boldsymbol{Y}_{14}^{\mathrm{T}}-\boldsymbol{Y}_{23}\\
* & * & * &
\begin{matrix}\tau_1\boldsymbol{X}_{144}+\tau\boldsymbol{X}_{244}-\boldsymbol{Y}_{24}-\boldsymbol{Y}_{24}^{\mathrm{T}}+\\ \tau_1\boldsymbol{K}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_i^{\mathrm{T}}\boldsymbol{R}(1-\beta)\bar{\boldsymbol{B}}_i\boldsymbol{K}_j+\\ \tau\boldsymbol{K}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_i^{\mathrm{T}}\boldsymbol{T}(1-\beta)\bar{\boldsymbol{B}}_i\boldsymbol{K}_j\end{matrix}
\end{bmatrix},$$

则网络系统(7)均方指数稳定.

证明　针对系统(7),设计 Lyapunov 泛函

$$V(t)=x^{\mathrm{T}}(t)\boldsymbol{P}x(t)+\int_{t-d}^{t}x^{\mathrm{T}}(s)\boldsymbol{Q}\,\mathrm{e}^{2\alpha(s-t)}x(s)\mathrm{d}s+\int_{-\tau_1}^{0}\int_{t-\theta}^{t}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}\mathrm{e}^{2\alpha(s-t)}\dot{x}(s)\mathrm{d}s\,\mathrm{d}\theta+$$
$$\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^{\mathrm{T}}(s)\boldsymbol{T}\mathrm{e}^{2\alpha(s-t)}\dot{x}(s)\mathrm{d}s\,\mathrm{d}\theta,$$

其中 $\boldsymbol{P},\boldsymbol{Q},\boldsymbol{R},\boldsymbol{T}$ 是正定矩阵.沿系统 (7),有

$$\dot{V}(t)+2\alpha V(t)=2x^{\mathrm{T}}(t)\boldsymbol{P}\dot{x}(t)+x^{\mathrm{T}}(t)\boldsymbol{Q}x(t)-x^{\mathrm{T}}(t-d)\boldsymbol{Q}\mathrm{e}^{-2\alpha d}x(t-d)+\tau_1\dot{x}^{\mathrm{T}}(t)\boldsymbol{R}\dot{x}(t)+$$
$$\tau\dot{x}(t)\boldsymbol{T}\dot{x}(t)+2\alpha x^{\mathrm{T}}(t)\boldsymbol{P}x(t)-\int_{t-\tau_1}^{t}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}\mathrm{e}^{2\alpha(s-t)}\dot{x}(s)\mathrm{d}s-\int_{t-\tau}^{t}\dot{x}^{\mathrm{T}}(s)\boldsymbol{T}\mathrm{e}^{2\alpha(s-t)}\dot{x}(s)\mathrm{d}s. \tag{9}$$

对任意的 $4n\times n$ 矩阵

$$\boldsymbol{N}=\begin{bmatrix}\boldsymbol{N}_1\\ \boldsymbol{N}_2\\ \boldsymbol{N}_3\end{bmatrix},\boldsymbol{M}=\begin{bmatrix}\boldsymbol{M}_1\\ \boldsymbol{M}_2\\ \boldsymbol{M}_3\end{bmatrix},$$

知道

$$0=\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{N}[x(t)-x(t-h_1(t))-\int_{t-h_1(t)}^{t}\dot{x}(s)\mathrm{d}s], \tag{10}$$

$$0=\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{M}[x(t)-x(t-h_2(t))\int_{t-h_2(t)}^{t}\dot{x}(s)\mathrm{d}s]. \tag{11}$$

由引理 2 和 (10)、(11)式，得到

$$0\leq 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{Y}_1[x(t)-x(t-h_1(t))]+\tau_1\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{X}_1\xi(t)\int_{t-\tau_1}^{t}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}\mathrm{e}^{2\alpha(s-t)}\dot{x}(s)\mathrm{d}s, \tag{12}$$

$$0\leq 2\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{Y}_2[x(t)-x(t-h_2(t))]+\tau_1\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{X}_2\xi(t)\int_{t-\tau_1}^{t}\dot{x}^{\mathrm{T}}(s)\boldsymbol{R}\mathrm{e}^{2\alpha(s-t)}\dot{x}(s)\mathrm{d}s, \tag{13}$$

把 (12)、(13)式代入(9)式，得到

$$\dot{V}(t)+2\alpha V(t)\leq\sum_{i}^{r}\sum_{j}^{r}h_i(z(t))h_j(z(t))\{x^{\mathrm{T}}(t)[\boldsymbol{P}\bar{\boldsymbol{A}}_i+\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{Q}+2\alpha\boldsymbol{P}]x(t)+2x^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{A}}_{di}x(t-$$
$$d)+2x^{\mathrm{T}}(t)\boldsymbol{P}\beta(t)\bar{\boldsymbol{B}}_i\boldsymbol{K}_jx(t-h_1(t))+2x^{\mathrm{T}}(t)\boldsymbol{P}(1-\beta(t))\bar{\boldsymbol{B}}_i\boldsymbol{K}_jx(t-h_2(t))-$$
$$x^{\mathrm{T}}(t-d)\boldsymbol{Q}\mathrm{e}^{-2\alpha d}x(t-d)+2\xi^{\mathrm{T}}(t)\boldsymbol{Y}_1[x(t)-x(t-h_1(t))]+\tau_1\xi^{\mathrm{T}}(t)X_1\xi(t)+$$
$$2\xi^{\mathrm{T}}(t)\boldsymbol{Y}_2[x(t)-x(t-h_2(t))]+\tau\xi^{\mathrm{T}}(t)X_2\xi(t)\}+\tau_1\dot{x}^{\mathrm{T}}(t)\boldsymbol{R}\dot{x}(t)+\tau\dot{x}^{\mathrm{T}}(t)\boldsymbol{T}\dot{x}(t). \tag{14}$$

由引理 1,有

$$\tau_1\dot{x}^{\mathrm{T}}(t)\boldsymbol{R}\dot{x}(t)\leq\tau_1\sum_{i}^{r}\sum_{j}^{r}\mu_i(z(t))\mu_j(z(t))\boldsymbol{\xi}^{\mathrm{T}}(t)\bar{\mathbf{A}}_{ij}^{\mathrm{T}}\boldsymbol{R}\bar{\boldsymbol{A}}_{ij}\xi(t) \tag{15}$$

和

$$\tau\dot{x}^{\mathrm{T}}(t)\boldsymbol{T}\dot{x}(t)\leq\tau\sum_{i}^{r}\sum_{j}^{r}\mu_i(z(t))\mu_j(z(t))\boldsymbol{\xi}^{\mathrm{T}}(t)\bar{\boldsymbol{A}}_{ij}^{\mathrm{T}}\boldsymbol{T}\bar{\boldsymbol{A}}_{ij}\xi(t), \tag{16}$$

把 (15)、(16)式代入(14)式,得到

$$E\{\dot{V}(t)+2\alpha V(t)\}\leq\sum_{i}^{r}\sum_{j}^{r}\mu_i(z(t))\mu_j(z(t))\boldsymbol{\xi}^{\mathrm{T}}(t)\boldsymbol{\Theta}\xi(t),$$

由矩阵不等式 (5),知道

$$E\{\dot{V}(t)\}<-2\alpha E\{V(t)\},$$

从而

$$E(t)<E\{V(0)\}\mathrm{e}^{-2\alpha t}\leq[\lambda_{\max}(\boldsymbol{P})+d\lambda_{\max}(\boldsymbol{Q})+$$
$$\tau_1\lambda_{\max}(\boldsymbol{R})\bar{\psi}^2+\tau\lambda_{\max}(\boldsymbol{T})\bar{\psi}^2]E\{\|\psi(t)\|^2\}\mathrm{e}^{-2\alpha t}, \tag{17}$$

显然

$$E\{V(t)\}\geq\lambda_{\max}(\boldsymbol{P})E\{\|x(t)\|^2\}. \tag{18}$$

由 (17)、(18)式,易得

$$E\{\|x(t)\|\} < \sqrt{\frac{\lambda_{\max}(\boldsymbol{P}) + d\lambda_{\max}(\boldsymbol{Q}) + \tau_1\lambda_{\max}(\boldsymbol{R})\bar{\psi}^2 + \tau\lambda_{\max}(\boldsymbol{T})\bar{\psi}^2}{\lambda_{\min}(\boldsymbol{P})}}E\{\|\psi(t)\|\}\mathrm{e}^{-\alpha t}. \tag{19}$$

由定义 1 和(19)式可知网络系统(7)均方指数稳定.

定理 2　假设 $\alpha > 0, 1 \geq \beta \geq 0$ 和 $i, j = 1, 2, \cdots, q$ 是给定的常数，针对系统(7)，如果存在正定矩阵 $\bar{\boldsymbol{P}}$，$\bar{\boldsymbol{Q}}, \bar{\boldsymbol{R}}, \bar{\boldsymbol{T}} \in \mathbf{R}^{n\times n}$，矩阵 $\bar{\boldsymbol{X}}_{1ij}, \bar{\boldsymbol{X}}_{2ij}, \bar{\boldsymbol{Y}}_{ij} \in \mathbf{R}^{n\times n}, \bar{\boldsymbol{K}}_j \in \mathbf{R}^{m\times n}$，满足线性矩阵不等式

$$\boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{\Xi}_{11} & \boldsymbol{\Xi}_{12} \\ * & \boldsymbol{\Xi}_{22} \end{bmatrix} < 0, \tag{20}$$

其中

$$\boldsymbol{\Xi}_{11} = \begin{bmatrix} \begin{matrix}\boldsymbol{A}_i\bar{\boldsymbol{P}} + \bar{\boldsymbol{P}}\boldsymbol{A}_i^{\mathrm{T}} + \bar{\boldsymbol{Q}} + 2\alpha\bar{\boldsymbol{P}} + \tau_1\bar{\boldsymbol{X}}_{111} + \\ \tau\bar{\boldsymbol{X}}_{211} + \bar{\boldsymbol{Y}}_{11} + \bar{\boldsymbol{Y}}_{11}^{\mathrm{T}} + \bar{\boldsymbol{Y}}_{21} + \bar{\boldsymbol{Y}}_{21}^{\mathrm{T}} + \varepsilon_1\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}}\end{matrix} & \begin{matrix}\boldsymbol{A}_{di}\bar{\boldsymbol{P}} + \tau_1\bar{\boldsymbol{X}}_{112} + \\ \tau\bar{\boldsymbol{X}}_{212} + \bar{\boldsymbol{Y}}_{12}^{\mathrm{T}} + \bar{\boldsymbol{Y}}_{22}^{\mathrm{T}}\end{matrix} & \begin{matrix}\beta\boldsymbol{B}_i\bar{\boldsymbol{K}}_j + \tau\bar{\boldsymbol{X}}_{113} + \tau\bar{\boldsymbol{X}}_{213} - \\ \bar{\boldsymbol{Y}}_{11} + \bar{\boldsymbol{Y}}_{13}^{\mathrm{T}} + \bar{\boldsymbol{Y}}_{23}^{\mathrm{T}}\end{matrix} & \begin{matrix}(1-\beta)\boldsymbol{B}_i\bar{\boldsymbol{K}}_j + \tau_1\bar{\boldsymbol{X}}_{114} + \\ \tau\bar{\boldsymbol{X}}_{214} + \bar{\boldsymbol{Y}}_{14}^{\mathrm{T}} - \bar{\boldsymbol{Y}}_{21} + \bar{\boldsymbol{Y}}_{24}^{\mathrm{T}}\end{matrix} \\ * & -\mathrm{e}^{-2\alpha d}\bar{\boldsymbol{Q}} + \tau_1\bar{\boldsymbol{X}}_{122} + \tau\bar{\boldsymbol{X}}_{222} & \tau_1\bar{\boldsymbol{X}}_{123} + \tau\bar{\boldsymbol{X}}_{223} - \bar{\boldsymbol{Y}}_{12} & \tau_1\bar{\boldsymbol{X}}_{124} + \tau\bar{\boldsymbol{X}}_{224} - \bar{\boldsymbol{Y}}_{22} \\ * & * & \tau_1\bar{\boldsymbol{X}}_{123} + \tau\bar{\boldsymbol{X}}_{223} - \bar{\boldsymbol{Y}}_{13} - \bar{\boldsymbol{Y}}_{13}^{\mathrm{T}} & \tau_1\bar{\boldsymbol{X}}_{134} + \tau\bar{\boldsymbol{X}}_{234} - \bar{\boldsymbol{Y}}_{14}^{\mathrm{T}} - \bar{\boldsymbol{Y}}_{23} \\ * & * & * & \tau_1\bar{\boldsymbol{X}}_{144} + \tau\bar{\boldsymbol{X}}_{244} - \bar{\boldsymbol{Y}}_{24} - \bar{\boldsymbol{Y}}_{24}^{\mathrm{T}} \end{bmatrix},$$

$$\boldsymbol{\Xi}_{12} = \begin{bmatrix} \tau_1\beta\bar{\boldsymbol{P}}\boldsymbol{A}_i^{\mathrm{T}} & \tau_1(1-\beta)\bar{\boldsymbol{P}}\boldsymbol{A}_i^{\mathrm{T}} & \tau\beta\bar{\boldsymbol{P}}\boldsymbol{A}_i^{\mathrm{T}} & \tau(1-\beta)\bar{\boldsymbol{P}}\boldsymbol{A}_i^{\mathrm{T}} & \bar{\boldsymbol{P}}\boldsymbol{E}_{i1}^{\mathrm{T}} & \tau_1\beta\bar{\boldsymbol{P}}\boldsymbol{E}_{i1}^{\mathrm{T}} & \tau_1(1-\beta)\bar{\boldsymbol{P}}\boldsymbol{E}_{i1}^{\mathrm{T}} & \tau\beta\bar{\boldsymbol{P}}\boldsymbol{E}_{i1}^{\mathrm{T}} & \tau(1-\beta)\bar{\boldsymbol{P}}\boldsymbol{E}_{i1}^{\mathrm{T}} \\ \tau_1\beta\bar{\boldsymbol{P}}\boldsymbol{A}_{di}^{\mathrm{T}} & \tau_1(1-\beta)\bar{\boldsymbol{P}}\boldsymbol{A}_{di}^{\mathrm{T}} & \tau\beta\bar{\boldsymbol{P}}\boldsymbol{A}_{di}^{\mathrm{T}} & \tau(1-\beta)\bar{\boldsymbol{P}}\boldsymbol{A}_{di}^{\mathrm{T}} & \bar{\boldsymbol{P}}\boldsymbol{E}_{i2}^{\mathrm{T}} & \tau_1\beta\bar{\boldsymbol{P}}\boldsymbol{E}_{i2}^{\mathrm{T}} & \tau_1(1-\beta)\bar{\boldsymbol{P}}\boldsymbol{E}_{i2}^{\mathrm{T}} & \tau\beta\bar{\boldsymbol{P}}\boldsymbol{E}_{i2}^{\mathrm{T}} & \tau(1-\beta)\bar{\boldsymbol{P}}\boldsymbol{E}_{i2}^{\mathrm{T}} \\ \tau_1\beta\bar{\boldsymbol{K}}_j^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}} & 0 & \tau\beta\bar{\boldsymbol{K}}_j^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}} & 0 & \beta\bar{\boldsymbol{K}}_j^{\mathrm{T}}\boldsymbol{E}_{i3}^{\mathrm{T}} & \tau_1\beta\bar{\boldsymbol{K}}_j^{\mathrm{T}}\boldsymbol{E}_{i3}^{\mathrm{T}} & 0 & \tau\beta\bar{\boldsymbol{K}}_j^{\mathrm{T}}\boldsymbol{E}_{i3}^{\mathrm{T}} & 0 \\ 0 & \tau_1(1-\beta)\bar{\boldsymbol{K}}_j^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}} & 0 & \tau(1-\beta)\bar{\boldsymbol{K}}_j^{\mathrm{T}}\boldsymbol{B}_i^{\mathrm{T}} & (1-\beta)\bar{\boldsymbol{K}}_j^{\mathrm{T}}\boldsymbol{E}_{i3}^{\mathrm{T}} & 0 & \tau_1(1-\beta)\bar{\boldsymbol{K}}_j^{\mathrm{T}}\boldsymbol{E}_{i3}^{\mathrm{T}} & 0 & \tau(1-\beta)\bar{\boldsymbol{K}}_j^{\mathrm{T}}\boldsymbol{E}_{i3}^{\mathrm{T}} \end{bmatrix},$$

$$\boldsymbol{\Xi}_{22} = \begin{bmatrix} -\tau_1\beta\bar{\boldsymbol{R}} + \varepsilon_2\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -\tau_1(1-\beta)\bar{\boldsymbol{R}} + \varepsilon_3\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\tau\beta\bar{\boldsymbol{T}} + \varepsilon_4\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\tau(1-\beta)\bar{\boldsymbol{T}} + \varepsilon_5\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1\boldsymbol{I} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_2\boldsymbol{I} & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_3\boldsymbol{I} & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_4\boldsymbol{I} & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_5\boldsymbol{I} \end{bmatrix},$$

选取控制器增益矩阵 $\boldsymbol{K}_1 = \bar{\boldsymbol{K}}_1\bar{\boldsymbol{P}}^{-1}, \boldsymbol{K}_2 = \bar{\boldsymbol{K}}_2\bar{\boldsymbol{P}}^{-1}$，则系统(7)是均方指数稳定的.

证明

$$\boldsymbol{\Theta} = \boldsymbol{\Theta}_0 + \alpha_1^{\mathrm{T}}\frac{1}{\tau_1\beta}\boldsymbol{R}^{-1}\alpha_1 + \alpha_2^{\mathrm{T}}\frac{1}{\tau_1(1-\beta)}\boldsymbol{R}^{-1}\alpha_2 + \alpha_3^{\mathrm{T}}\frac{1}{\tau\beta}\boldsymbol{T}^{-1}\alpha_3 + \alpha_4^{\mathrm{T}}\frac{1}{\tau(1-\beta)}\boldsymbol{T}^{-1}\alpha_4 < 0,$$

其中

$$\boldsymbol{\Theta}_0 = \begin{bmatrix} \begin{matrix}\boldsymbol{P}\bar{\boldsymbol{A}}_i + \bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{Q} + 2\alpha\boldsymbol{P} + \tau_1\boldsymbol{X}_{111} + \\ \tau\boldsymbol{X}_{211} + \boldsymbol{Y}_{11} + \boldsymbol{Y}_{11}^{\mathrm{T}} + \boldsymbol{Y}_{21} + \boldsymbol{Y}_{21}^{\mathrm{T}}\end{matrix} & \begin{matrix}\boldsymbol{P}\bar{\boldsymbol{A}}_{di} + \tau_1\boldsymbol{X}_{112} + \\ \tau\boldsymbol{X}_{212} + \boldsymbol{Y}_{12}^{\mathrm{T}} + \boldsymbol{Y}_{22}^{\mathrm{T}}\end{matrix} & \begin{matrix}\boldsymbol{P}\beta\bar{\boldsymbol{B}}_i\boldsymbol{K}_j + \tau_1\boldsymbol{X}_{113} + \\ \tau\boldsymbol{X}_{213} - \boldsymbol{Y}_{11} + \boldsymbol{Y}_{13}^{\mathrm{T}} + \boldsymbol{Y}_{23}^{\mathrm{T}}\end{matrix} & \begin{matrix}\boldsymbol{P}(1-\beta)\bar{\boldsymbol{B}}_i\boldsymbol{K}_j + \tau_1\boldsymbol{X}_{114} + \\ \tau\boldsymbol{X}_{214} + \boldsymbol{Y}_{14}^{\mathrm{T}} - \boldsymbol{Y}_{21} + \boldsymbol{Y}_{24}^{\mathrm{T}}\end{matrix} \\ * & -\mathrm{e}^{-2\alpha d}\boldsymbol{Q} + \tau_1\boldsymbol{X}_{122} + \tau\boldsymbol{X}_{222} & \tau_1\boldsymbol{X}_{123} + \tau\boldsymbol{X}_{223} - \boldsymbol{Y}_{12} & \tau_1\boldsymbol{X}_{124} + \tau\boldsymbol{X}_{224} - \boldsymbol{Y}_{22} \\ * & * & \tau_1\boldsymbol{X}_{133} + \tau\boldsymbol{X}_{233} - \boldsymbol{Y}_{13} - \boldsymbol{Y}_{13}^{\mathrm{T}} & \tau_1\boldsymbol{X}_{134} + \tau\boldsymbol{X}_{234} - \boldsymbol{Y}_{14}^{\mathrm{T}} - \boldsymbol{Y}_{23} \\ * & * & * & \tau_1\boldsymbol{X}_{144} + \tau\boldsymbol{X}_{244} - \boldsymbol{Y}_{24} - \boldsymbol{Y}_{24}^{\mathrm{T}} \end{bmatrix},$$

$$\alpha_1 = \begin{bmatrix}\tau_1\beta\boldsymbol{R}\bar{\boldsymbol{A}}_i & \tau_1\beta\boldsymbol{R}\bar{\boldsymbol{A}}_{di} & \tau_1\beta\boldsymbol{R}\bar{\boldsymbol{B}}_i\boldsymbol{K}_j & 0\end{bmatrix},$$

$$\alpha_2 = \begin{bmatrix}\tau_1(1-\beta)\mathbf{R}\bar{\boldsymbol{A}}_i & \tau_1(1-\beta)\boldsymbol{R}\bar{\boldsymbol{A}}_{di} & 0 & \tau_1(1-\beta)\boldsymbol{R}\bar{\boldsymbol{B}}_i\boldsymbol{K}_j\end{bmatrix},$$

$$\alpha_3 = \begin{bmatrix}\tau\beta\boldsymbol{T}\bar{\boldsymbol{A}}_i & \tau\beta\boldsymbol{T}\bar{\boldsymbol{A}}_{di} & \tau\beta\boldsymbol{T}\bar{\boldsymbol{B}}_i\boldsymbol{K}_j & 0\end{bmatrix},$$

$$\alpha_4 = \begin{bmatrix}\tau(1-\beta)\boldsymbol{R}\bar{\boldsymbol{A}}_i & \tau(1-\beta)\boldsymbol{T}\bar{\boldsymbol{A}}_{di} & 0 & \tau(1-\beta)\boldsymbol{T}\bar{\boldsymbol{B}}_i\boldsymbol{K}_j\end{bmatrix}.$$

由引理 3，知道不等式 $\boldsymbol{\Theta} < 0$ 等价于

$$\begin{bmatrix} \begin{matrix}\boldsymbol{P}\bar{\boldsymbol{X}}_i+\bar{\boldsymbol{X}}_i^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{Q}+\\ 2\alpha\boldsymbol{P}+\tau_1\boldsymbol{X}_{111}+\\ \tau\boldsymbol{X}_{211}+\boldsymbol{Y}_{11}\boldsymbol{Y}_{11}^{\mathrm{T}}+\\ \boldsymbol{Y}_{21}+\boldsymbol{Y}_{21}^{\mathrm{T}}\end{matrix} & \begin{matrix}\boldsymbol{P}\bar{\boldsymbol{A}}_{di}+\\ \tau\boldsymbol{X}_{112}\tau\boldsymbol{X}_{212}+\\ \boldsymbol{Y}_{12}^{\mathrm{T}}+\boldsymbol{Y}_{22}^{\mathrm{T}}\end{matrix} & \begin{matrix}\boldsymbol{P}\beta\bar{\boldsymbol{B}}_i\boldsymbol{K}_j+\\ \tau_1\boldsymbol{X}_{113}+\tau\boldsymbol{X}_{213}-\\ \boldsymbol{Y}_{11}+\boldsymbol{Y}_{13}^{\mathrm{T}}\boldsymbol{Y}_{23}^{\mathrm{T}}\end{matrix} & \begin{matrix}\boldsymbol{P}(1-\beta)\bar{\boldsymbol{B}}_i\boldsymbol{K}_j+\\ \tau_1\boldsymbol{X}_{114}+\tau\boldsymbol{X}_{214}+\\ \boldsymbol{Y}_{14}^{\mathrm{T}}-\boldsymbol{Y}_{21}+\boldsymbol{Y}_{24}^{\mathrm{T}}\end{matrix} & \tau_1\beta\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{R} & \tau_1(1-\beta)\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{R} & \tau\beta\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{T} & \tau(1-\beta)\bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{T} \\ * & \begin{matrix}-\mathrm{e}^{-2\alpha d}\boldsymbol{Q}+\\ \tau_1\boldsymbol{X}_{122}+\tau\boldsymbol{X}_{222}\end{matrix} & \begin{matrix}\tau_1\boldsymbol{X}_{123}+\\ \tau\boldsymbol{X}_{223}-\boldsymbol{Y}_2\end{matrix} & \begin{matrix}\tau_1\boldsymbol{X}_{124}+\\ \tau\boldsymbol{X}_{224}-\boldsymbol{Y}_{22}\end{matrix} & \tau_1\beta\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}\boldsymbol{R} & \tau_1(1-\beta)\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}\boldsymbol{R} & \tau\beta\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}\boldsymbol{T} & \tau(1-\beta)\bar{\boldsymbol{A}}_{di}^{\mathrm{T}}\boldsymbol{T} \\ * & * & \begin{matrix}\tau_1\boldsymbol{X}_{133}+\tau\boldsymbol{X}_{233}-\\ \boldsymbol{Y}_{13}-\boldsymbol{Y}_{13}^{\mathrm{T}}\end{matrix} & \begin{matrix}\tau_1\boldsymbol{X}_{134}+\tau\boldsymbol{X}_{234}-\\ \boldsymbol{Y}_{14}-\boldsymbol{Y}_{23}^{\mathrm{T}}\end{matrix} & \tau_1\beta\boldsymbol{K}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_i^{\mathrm{T}}P\boldsymbol{R} & 0 & \tau\beta\boldsymbol{K}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_i^{\mathrm{T}}\boldsymbol{T} & 0 \\ * & * & * & \begin{matrix}\tau_1\boldsymbol{X}_{144}+\tau\boldsymbol{X}_{244}-\\ \boldsymbol{Y}_{24}-\boldsymbol{Y}_{24}^{\mathrm{T}}\end{matrix} & 0 & \begin{matrix}\tau(1-\\ \beta)\boldsymbol{K}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_i^{\mathrm{T}}\boldsymbol{R}\end{matrix} & 0 & \begin{matrix}\tau(1-\\ \beta)\boldsymbol{K}_j^{\mathrm{T}}\bar{\boldsymbol{B}}_i^{\mathrm{T}}\boldsymbol{T}\end{matrix} \\ * & * & * & * & -\tau_1\beta\boldsymbol{R} & 0 & 0 & 0 \\ * & * & * & * & * & -\tau_1(1-\beta)\boldsymbol{R} & 0 & 0 \\ * & * & * & * & * & * & -\tau\beta\boldsymbol{T} & 0 \\ * & * & * & * & * & * & * & -\tau(1-\beta)\boldsymbol{T} \end{bmatrix} < 0.$$

在上述不等式两边分别左乘和右乘以矩阵

$$\mathrm{diag}\{\boldsymbol{P}^{-1}\quad \boldsymbol{P}^{-1}\quad \boldsymbol{P}^{-1}\quad \boldsymbol{P}^{-1}\quad \boldsymbol{R}^{-1}\quad \boldsymbol{R}^{-1}\quad \boldsymbol{T}^{-1}\quad \boldsymbol{T}^{-1}\quad \boldsymbol{I}\quad \boldsymbol{I}\quad \boldsymbol{I}\quad \boldsymbol{I}\quad \boldsymbol{I}\},$$

并令

$$\bar{\boldsymbol{P}}=\boldsymbol{P}^{-1},\bar{\boldsymbol{Q}}=\boldsymbol{P}^{-1}\boldsymbol{Q}\boldsymbol{P}^{-1},\bar{\boldsymbol{K}}_j=\boldsymbol{K}_j\boldsymbol{P}^{-1},\bar{\boldsymbol{X}}_{ijk}=\boldsymbol{P}^{-1}\boldsymbol{X}_{ijk}\boldsymbol{P}^{-1},\bar{\boldsymbol{Y}}_{ij}=\boldsymbol{P}^{-1}\boldsymbol{Y}_{ij}\boldsymbol{P}^{-1},\bar{\boldsymbol{R}}=\boldsymbol{R}^{-1},\bar{\boldsymbol{T}}=\boldsymbol{T}^{-1},$$

可知不等式(20)等价于(8)式.因此,在控制器(3)的作用下,网络系统(7)指数稳定.

3 系统仿真

在网络系统(7)中选取 $\boldsymbol{A}_1=\begin{bmatrix}-1 & 2\\ 0 & -1\end{bmatrix}$,$\boldsymbol{A}_2=\begin{bmatrix}-2 & 0\\ 1 & -1\end{bmatrix}$,$\boldsymbol{A}_{d1}=\begin{bmatrix}-0.1 & 0.2\\ 0 & -0.2\end{bmatrix}$,$\boldsymbol{A}_{d2}=\begin{bmatrix}-0.2 & 0\\ 0.1 & -0.1\end{bmatrix}$,$\boldsymbol{D}=\begin{bmatrix}0.2\\ 0.1\end{bmatrix}$,$\boldsymbol{B}_1=\begin{bmatrix}0\\ 1\end{bmatrix}$,$\boldsymbol{B}_2=\begin{bmatrix}0.1\\ 0.2\end{bmatrix}$,$\tau=1$,$\tau_1=0.5$,$\beta=0.5$,$\alpha=0.1$,$d=0.1$,求解线性矩阵不等式(20),得到模糊控制器增益矩阵

$$\boldsymbol{K}_1=\bar{\boldsymbol{K}}_1\boldsymbol{P}^{-1}=[-7.562\,8\quad 2.368\,7],\boldsymbol{K}_2=\bar{\boldsymbol{K}}_2\bar{\boldsymbol{P}}^{-1}=[12.298\,4\quad -4.672\,3].$$

选取随机变量 $\beta(t)$ 如图 1.

选取初始条件为 $x(t)=\begin{bmatrix}-3\\ 4\end{bmatrix}$ 则选取模糊控制器(3),系统状态仿真如图 2.由图 2 可知,系统状态在 8 s内收敛到 0 状态,而且呈指数收敛趋势.

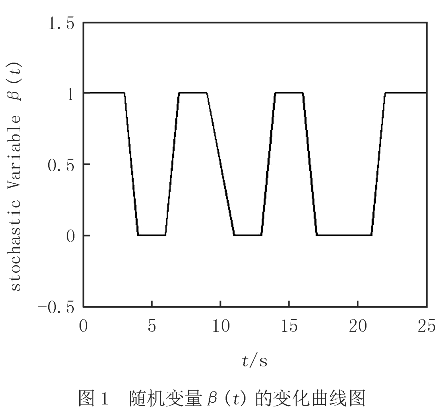

图 1 随机变量 $\beta(t)$ 的变化曲线图

Fig.1 Curve of the stochastic Variable $\beta(t)$

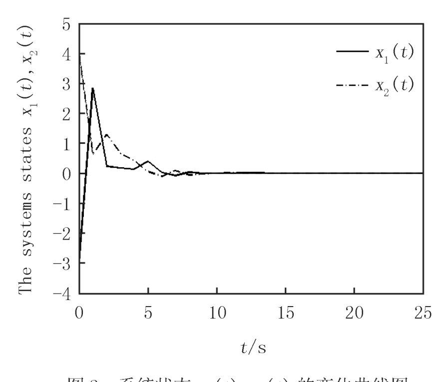

图 2 系统状态 $x_1(t),x_2(t)$ 的变化曲线图

Fig.2 Curves of the system states $x_1(t),x_2(t)$

4 结 论

本文在假设网络时延具有 Bernoulli 分布的统计特性基础上,建立网络系统的 T-S 模糊随机模型.通过构造带有网络时延的 Lyapunov 函数,得到系统指数稳定条件,降低了充分条件的保守性.然后给出了模糊控

制器增益矩阵的计算方法,并通过数值计算和计算机仿真说明此方法简单、可行.

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Exponential stability control for a class of nonlinear stochastic networked control systems

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Abstract: The problem of mean square exponential stability control for a class of nonlinear stochastic networked control systems is considered in this paper. By inserting a point into the time-varying interval of network-induced delay, the network-induced delay is transformed into a random variable satisfying the interval Bernoulli distribution. According to the values of random variables in different intervals, a new model of the networked control systems is established by using T-S fuzzy method. Then the linear matrix inequality method is applied to the processing of the new model. The delay dependence exponential stability condition is obtained, and the design method of fuzzy controller is given. The numerical calculation and simulation of specific networked systems are carried out.

Keywords: networked systems; exponential stability; stochastic systems; delay

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