具有恐惧效应和食饵避难的时滞捕食模型的稳定性

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摘　要:考虑恐惧效应和食饵避难,研究了具有阶段性结构和时滞的 Crowley-Martin 型捕食模型.首先分析了平衡点的存在性;接着通过对特征方程根的讨论以及构造 Lyapunov 函数得到了边界平衡点满足局部和全局渐近稳定性的条件;然后,研究了时滞对内部平衡点稳定性的影响,分析了系统在内部平衡点处 Hopf 分支的存在性;最后,通过 MATLAB 数值模拟对结果进行了验证.

关键词:恐惧效应;食饵避难;时滞;稳定性;Hopf 分支

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近年来,很多捕食模型的研究都考虑了食饵避难[1-5],这种进化过程中食饵对捕食者所形成的集体抵御,拟态保护等行为能够有效降低捕获率.此外,研究还表明,食饵对捕食者的恐惧情绪也是影响种群数量变化的重要因素,文献[6-10]具体分析了具有恐惧效应的捕食模型,其中文献[10]首次同时考虑了食饵避难和恐惧效应.文献[11]研究了功能性反应函数为 Beddington-DeAngelis 型的具有食饵避难和阶段性结构的模型,模型如下:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = x(r-ax) - \dfrac{\mu(1-m)xy_2}{1+b(1-m)x+cy_2}, \\[2mm] \dfrac{\mathrm{d}y_1}{\mathrm{d}t} = \dfrac{\eta\mu(1-m)x(t-\tau)y_2(t-\tau)}{1+b(1-m)x(t-\tau)+cy_2(t-\tau)} - \beta y_1 - dy_1, \\[2mm] \dfrac{\mathrm{d}y_2}{\mathrm{d}t} = \beta y_1 - ey_2, \end{cases} \tag{1}$$

其中 x,y_1,y_2 分别是食饵,幼年捕食者和成年捕食者,m 为食饵避难比例,τ 是成年捕食者的消化时滞.目前,具有 Crowley-Martin 功能反应的捕食系统得到了广泛关注[12-14],Crowley-Martin 功能反应既与食饵数量相关又与捕食者数量相关,同时考虑了捕食者之间的相互作用,相较 Beddington-DeAngelis 型功能反应,更符合生物学实际.因此,本文在模型的基础上,考虑加入恐惧效应并使用 Crowley-Martin 型功能反应函数,建立新的模型并研究其动力学性态.

1　模型的建立

建立的模型如下:

$$\begin{cases}\dfrac{\mathrm{d}x}{\mathrm{d}t}=\dfrac{rx}{1+ky_2}-d_0x-cx^2-\dfrac{\alpha(1-m)xy_2}{1+a(1-m)x+by_2+ab(1-m)xy_2},\\[3mm]\dfrac{\mathrm{d}y_1}{\mathrm{d}t}=\dfrac{\beta\alpha(1-m)x(t-\tau)y_2(t-\tau)}{1+a(1-m)x(t-\tau)+by_2(t-\tau)+ab(1-m)x(t-\tau)y_2(t-\tau)}-ny_1-d_1y_1,\\[3mm]\dfrac{\mathrm{d}y_2}{\mathrm{d}t}=ny_1-d_2y_2,\end{cases}\tag{2}$$

其中 x,y_1,y_2 分别表示食饵,幼年捕食者和成年捕食者在 t 时刻的种群密度. r 是未受捕食者影响的食饵出生率. k 代表食饵对成年捕食者的恐惧因子. d_0,d_1 和 d_2 分别表示食饵,幼年捕食者和成年捕食者的自然死亡率. c 是食饵的种内竞争因子, $m\in[0,1)$ 是 t 时刻食饵避难的比例. a,b,α 分别代表了成年捕食者的处理时间,相互作用的强度以及对食饵的捕获率. β 代表捕食转化率, τ 代表成年捕食者的消化时滞, n 代表幼年捕食者成长为成年捕食者的比例. 系统的初始条件满足:

$$x(\theta)=\varphi_1(\theta),y_1(\theta)=\varphi_2(\theta),y_2(0)=\varphi_3(\theta),$$

其中 $\varphi_1(\theta)\geqslant 0,\theta\in[-\tau,0],\varphi_1(1)>0(i=1,2,3),(\varphi_1(\theta),\varphi_2(\theta),\varphi_3(\theta))\in\mathscr{C}([-\tau,0],\mathbf{R}_+^3)$.

定义范数 $\|\varphi\|=\sup\limits_{-\tau\leqslant\theta\leqslant 0}\{|\varphi_1(\theta)|,|\varphi_2(\theta)|,|\varphi_3(\theta)|\}$,其中 $\varphi=(\varphi_1,\varphi_2,\varphi_3)$ 是连续映射, $\varphi:[-\tau,0]\rightarrow\mathbf{R}_+^3$,可知 $\mathscr{C}([-\tau,0],\mathbf{R}_+^3)$ 构成 Banach 空间.

2　平衡点的存在性

系统的平衡点如下:

(1)系统(2)始终存在一个平凡平衡点 $E_0=(0,0,0)$.

(2)若 $r>d_0$ 时,系统(2)存在一个边界平衡点 $E_1=(\dfrac{r-d_0}{c},0,0)$.

(3)内部平衡点 $E^*=(x^*,y_1^*,y_2^*)$,其中 x^*,y_1^* 和 y_2^* 是如下方程的解

$$F_1(x,y_2):=\dfrac{r}{1+ky_2}-d_0-cx-\dfrac{\alpha(1-m)y_2}{1+a(1-m)x+by_2+ab(1-m)xy_2}=0,\tag{3}$$

$$\dfrac{\beta\alpha(1-m)xy_2}{1+a(1-m)x+by_2+ab(1-m)xy_2}-ny_1-d_1y_1=0,\tag{4}$$

$$ny_1-d_2y_2=0.\tag{5}$$

由方程(5)可得 $y_1=\dfrac{d_2}{n}y_2$,将 $y_1=\dfrac{d_2}{n}y_2$ 代入(4)式,有

$$F_2(x,y_2):=\dfrac{\beta\alpha(1-m)x}{1+a(1-m)x+by_2+ab(1-m)xy_2}-d_2-\dfrac{d_1d_2}{n}=0.\tag{6}$$

由方程(3)可得:

(i)若 $x=0$,则有

$$y_2^2+h_1y_2+h_2=0,\tag{7}$$

其中

$$h_1=\dfrac{\alpha(1-m)+d_0(k+b)-rb}{k(\alpha(1-m)+d_0b)},h_2=\dfrac{d_0-r}{k(\alpha(1-m)+d_0b)}.$$

显然,若 $r>d_0$,方程(7)只有一个正根 $y_2^{(1)}$.

(ii)若 $y_2=0$,则 $x^{(1)}=\dfrac{r-d_0}{c}$,且 $x^{(1)}>0$ 时,有 $r>d_0$.

(iii)易得 $\dfrac{\mathrm{d}y_2}{\mathrm{d}x}=-\dfrac{F_{1x}}{F_{1y_2}}=\dfrac{\dfrac{\alpha a(1-m)^2(1+by_2)y_2}{(1+a(1-m)x+by_2+ab(1-m)xy_2)^2}-c}{\dfrac{rk}{(1+ky_2)^2}+\dfrac{\alpha(1-m)(1+a(1-m)x)}{(1+a(1-m)x+by_2+ab(1-m)xy_2)^2}}.$

当 $\dfrac{\mathrm{d}y_2}{\mathrm{d}x} < 0$ 时,有

$$\frac{\alpha a(1-m)^2(1+by_2)y_2}{(1+a(1-m)x+by_2+ab(1-m)xy_2)^2} < c. \tag{8}$$

将(3)式代入(8)式,可得

$$l_1 xy_2 + l_2 x + l_3 y_2 + l_4 > 0, \tag{9}$$

其中

$$l_1 = 2kac(1-m), l_2 = 2a(1-m),$$
$$l_3 = k(d_0 a(1-m)+c), l_4 = d_0 a(1-m)+c-ra(1-m).$$

对于所有 $x > 0, y_2 > 0$,若 $a(r-d_0)(1-m) \leqslant c$,则(9)式成立,进而有 $\dfrac{\mathrm{d}y_2}{\mathrm{d}x} < 0$.

同样地,由方程(6)可得:

(i)若 $y_2 = 0$,则 $x^{(2)} = \dfrac{d_2(n+d_1)}{(1-m)(n\beta\alpha - d_2 a(n+d_1))}$,且 $x^{(2)} > 0$ 时,有 $n\beta\alpha > d_2 a(n+d_1)$.当 $x^{(2)} < x^{(1)}$ 时,有 $d_2 c(n+d_1) < (r-d_0)(n\beta\alpha - d_2 a(n+d_1))(1-m)$.

(ii)易得 $\dfrac{\mathrm{d}y_2}{\mathrm{d}x} = -\dfrac{F_{2x}}{F_{2y_2}} = \dfrac{\beta\alpha(1-m)(1+by_2)}{\beta\alpha b(1-m)(1+a(1-m)x)x} > 0$.

因此,若以下条件满足,(3)和(6)式的图像在第一象限有唯一交点 (x^*, y_2^*),将 y_2^* 的值代入(5)式可以得到 y_1^*.即存在唯一内部平衡点 $E^* = (x^*, y_1^*, y_2^*)$.

(R1) $r > d_0$;

(R2) $n\beta\alpha > d_2 a(n+d_1)$;

(R3) $a(r-d_0)(1-m) \leqslant c$;

(R4) $d_2 c(n+d_1) < (r-d_0)(n\beta\alpha - d_2 a(n+d_1))(1-m)$.

3 边界平衡点的稳定性

定理 1 若 $r < d_0$,则平凡平衡点 E_0 是局部渐近稳定的;若 $r > d_0$,则平凡平衡点 E_0 不稳定.

证明 平凡平衡点 E_0 处的特征方程为

$$(\lambda - r + d_0)(\lambda + n + d_1)(\lambda + d_2) = 0. \tag{10}$$

方程(10)有 3 个根 $\lambda_1 = r - d_0, \lambda_2 = -n - d_1 < 0$ 和 $\lambda_3 = -d_2 < 0$.显然,当 $r < d_0$ 时,平凡平衡点 E_0 是局部渐近稳定的.当 $r > d_0$ 时,E_0 是不稳定的.

定理 2 若(R1)和(R2)成立,当 $d_2 c(n+d_1) > (r-d_0)(n\beta\alpha - d_2 a(n+d_1))(1-m)$ 时,边界平衡点 E_1 是局部渐近稳定的.

证明 边界平衡点 E_1 处的特征方程为

$$(\lambda + r - d_0)\left[(\lambda + n + d_1)(\lambda + d_2) - \frac{n\beta\alpha(r-d_0)(1-m)}{c + a(r-d_0)(1-m)}\mathrm{e}^{-\lambda\tau}\right] = 0. \tag{11}$$

若 $r > d_0$,方程 $\lambda + r - d_0 = 0$ 有一个负根,则方程(11)的其他根由

$$\lambda^2 + k_1\lambda + k_2 + k_3\mathrm{e}^{-\lambda\tau} = 0 \tag{12}$$

决定,其中

$$k_1 = n + d_1 + d_2, k_2 = d_2(n+d_1), k_3 = -\frac{n\beta\alpha(r-d_0)(1-m)}{c + a(r-d_0)(1-m)}.$$

当 $\tau = 0$ 时,方程(12)转化为

$$\lambda^2 + k_1\lambda + k_2 + k_3 = 0. \tag{13}$$

由 Routh-Hurwitz 判别定理,当 $k_2 + k_3 > 0$ 时,边界平衡点 E_1 是局部渐近稳定的.若(R4)成立,方程(13)至

少有一个正实根,则 E_1 是不稳定的.

当 $\tau > 0$ 时,讨论方程(12)纯虚根的存在性.若 $i\omega_1(\omega_1 > 0)$ 是方程(12)的一个解,当且仅当 ω_1 满足

$$-\omega_1^2 + ik_1\omega_1 + k_2 + k_3(\cos\tau\omega_1 - i\sin\tau\omega_1) = 0.$$

分离实部与虚部,可得

$$\begin{cases} k_3\sin\tau\omega_1 = k_1\omega_1, \\ k_3\cos\tau\omega_1 = \omega_1^2 - k_2, \end{cases} \tag{14}$$

等式两边同时平方再相加可得

$$\omega_1^4 + (k_1^2 - 2k_2)\omega_1^2 + k_2^2 - k_3^2 = 0. \tag{15}$$

注意到

$$k_1^2 - 2k_2 = (n + d_1)^2 + d_2^2 > 0,$$

$$k_2 - k_3 = d_2(n + d_1) + \frac{n\beta\alpha(r - d_0)(1 - m)}{c + a(r - d_0)(1 - m)} > 0.$$

若 $k_2 + k_3 > 0$,可得 $k_2^2 - k_3^2 > 0$ 并且可知对于 $\tau \geq 0$,方程(12)所有的根都具有负实部,则边界平衡点 E_1 是局部渐近稳定的.

定理 3 若(R1)成立,当 $d_2c(n + d_1) > (r - d_0)(n\beta rk + n\beta\alpha(1 - m) - d_2a(n + d_1)(1 - m))$ 时,边界平衡点 E_1 是全局渐近稳定的.

证明 构造一个 Lyapunov 函数

$$V(t) = x - x_0 - x_0\ln\frac{x}{x_0} + \xi_1 y_1 + \xi_2 y_2 + \xi_1\beta\alpha(1 - m)\int_{t-\tau}^t \frac{x(s)y_2(s)}{(1 + a(1 - m)x(t - \tau))(1 + by_2(t - \tau))}ds,$$

其中

$$x_0 = \frac{r - d_0}{c}, \xi_1 = \frac{1 + a(1 - m)x_0}{\beta}, \xi_2 = \frac{rkx_0 + \alpha(1 - m)x_0}{d_2}.$$

计算 $V(t)$ 沿系统(2)对时间 t 的全导数,可得

$$\dot{V}(t) = \left(1 - \frac{x_0}{x}\right)\left(\frac{rx}{1 + ky_2} - d_0x - cx^2 - \frac{\alpha(1 - m)xy_2}{(1 + a(1 - m)x)(1 + by_2)}\right) +$$

$$\xi_1\left(\frac{\beta\alpha(1 - m)x(t - \tau)y_2(t - \tau)}{(1 + a(1 - m)x(t - \tau))(1 + by_2(t - \tau))} - ny_1 - d_1y_1\right) + \xi_2(ny_1 - d_2y_2) +$$

$$\frac{\xi_1\beta\alpha(1 - m)xy_2}{(1 + a(1 - m)x)(1 + by_2)} - \frac{\xi_1\beta\alpha(1 - m)x(t - \tau)y_2(t - \tau)}{(1 + a(1 - m)x(t - \tau))(1 + by_2(t - \tau))} =$$

$$-c(x - x_0)^2 - \frac{rkxy_2}{1 + ky_2} - \frac{\alpha b(1 - m)x_0y_2^2}{(1 + a(1 - m)x)(1 + by_2)} - \frac{rk^2x_0y_2^2}{1 + ky_2} -$$

$$\frac{\alpha ab(1 - m)^2x_0xy_2^2}{(1 + a(1 - m)x)(1 + by_2)} - [(n + d_1)\xi_1 - n\xi_2]y_1.$$

记 $\mathcal{M}$ 为包含于 $\{(x, y_1, y_2) \in \mathbf{R}_+^3: x > 0 \mid \dot{V}(t) = 0\}$ 的最大不变集.若 $(n + d_1)\xi_1 > n\xi_2$,即

$$d_2c(n + d_1) > (r - d_0)(n\beta rk + n\beta\alpha(1 - m) - d_2a(n + d_1)(1 - m)),$$

则对任意 $x > 0, y_1 \geq 0, y_2 \geq 0$ 都有 $\dot{V}(t) \leq 0$.又发现当且仅当 $\mathcal{M} = \{E_1\}$ 时,$\dot{V}(t) = 0$ 才成立.通过 LaSalle 不变集原理,E_1 全局渐近稳定.

4 内部平衡点的稳定性

边界平衡点 $E^* = (x^*, y_1^*, y_2^*)$ 处的特征方程为

$$\lambda^3 + p_1\lambda^2 + p_2\lambda + p_3 + (q_1\lambda + q_2)e^{-\lambda\tau} = 0, \tag{16}$$

其中

$$p_1 = n + d_1 + d_2 + A, p_2 = d_2(n + d_1) + (n + d_1 + d_2)A, p_3 = d_2(n + d_1)A, q_1 = -B, q_2 = CD - AB,$$

$$A = cx^* - \frac{\alpha a(1-m)^2(1+by_2^*)x^*y_2^*}{(1+a(1-m)x^*+by_2^*+ab(1-m)x^*y_2^*)^2},$$

$$B = \frac{n\beta\alpha(1-m)(1+a(1-m)x^*)x^*}{(1+a(1-m)x^*+by_2^*+ab(1-m)x^*y_2^*)^2},$$

$$C = \frac{n\beta\alpha(1-m)(1+by_2^*)y_2^*}{(1+a(1-m)x^*+by_2^*+ab(1-m)x^*y_2^*)^2},$$

$$D = \frac{rkx^*}{(1+ky_2^*)^2} + \frac{\alpha(1-m)(1+a(1-m)x^*)x^*}{(1+a(1-m)x^*+by_2^*+ab(1-m)x^*y_2^*)^2}.$$

当 $\tau=0$ 时,方程(16)转化为

$$\lambda^3 + p_1\lambda^2 + (p_2+q_1)\lambda + p_3 + q_2 = 0. \tag{17}$$

由(6)式可得

$$B = \frac{n\beta\alpha(1-m)(1+a(1-m)x^*)x^*}{(1+a(1-m)x^*+by_2^*+ab(1-m)x^*y_2^*)^2} = \frac{d_2(n+d_1)}{1+by_2^*}.$$

则

$$p_1 = n+d_1+d_2+A, \quad p_2+q_1 = d_2(n+d_1)\left(1-\frac{1}{1+by_2^*}\right)+(n+d_1+d_2)A,$$

$$p_3+q_2 = d_2(n+d_1)\left(1-\frac{1}{1+by_2^*}\right)A+CD,$$

$$p_1(p_2+q_1)-(p_3+q_2) = d_2(n+d_1)(n+d_1+d_2)\left(1-\frac{1}{1+by_2^*}\right)+$$

$$(n+d_1+d_2)(n+d_1+d_2+A)A-CD.$$

若 $p_1(p_2+q_1) > p_3+q_2$ 和(R1)~(R4)成立,由 Routh-Hurwitz 定理,可知内部平衡点 E^* 是局部渐近稳定的.

当 $\tau>0$ 时,讨论方程(16)纯虚根的存在性.若 $i\omega(\omega>0)$ 是方程(16)的一个解,当且仅当 ω 满足

$$-i\omega^3 - p_1\omega^2 + ip_2\omega + p_3 + (iq_1\omega+q_2)(\cos\tau\omega - i\sin\tau\omega) = 0.$$

分离实部与虚部,可得

$$\begin{cases} q_1\omega\cos\tau\omega - q_2\sin\tau\omega = \omega^3 - p_2\omega, \\ q_1\omega\sin\tau\omega + q_2\cos\tau\omega = p_1\omega^2 - p_3, \end{cases} \tag{18}$$

等式两边同时平方再相加可得

$$\omega^6 + (p_1^2-2p_2)\omega^4 + (p_2^2-2p_1p_3-q_1^2)\omega^2 + p_3^2-q_2^2 = 0, \tag{19}$$

其中

$$p_1^2-2p_2 = (n+d_1)^2+d_2^2+A^2 > 0,$$

$$p_2^2-2p_1p_3-q_1^2 = d_2^2(n+d_1)^2\left(1-\frac{1}{(1+by_2^*)^2}\right)+[d_2^2+(n+d_1)^2]A^2 > 0,$$

$$p_3^2-q_2^2 = (p_3+q_2)(p_3-q_2).$$

显然,若 $p_3 > q_2$ 且(R1)~(R4)成立,则 $p_3^2-q_2^2 > 0$,这意味着方程(19)没有正实根.因此,对于 $\tau\geq0$,内部平衡点 E^* 是局部渐近稳定的.注意到

$$p_1(p_2+q_1) = d_2(n+d_1)(n+d_1+d_2+A)\left(1-\frac{1}{1+by_2^*}\right)+(n+d_1+d_2)A^2+$$

$$n(n+d_1)^2A+d_2^2A+2d_2(n+d_1)A > 2d_2(n+d_1)A = 2p_3.$$

若 $p_3 > q_2$,则 $p_1(p_2+q_1) > 2p_3 > p_3+q_2$.

若 $p_3 < q_2$,则 $p_3^2-q_2^2 < 0$,存在唯一的正根 ω_0 满足(19)式.

由(18)式可得

$$\cos \tau \omega_0 = \frac{q_1 \omega_0^4 + (p_1 q_2 - p_2 q_1)\omega_0^2 - p_3 q_2}{q_1^2 \omega_0^2 + q_2^2}.$$

记

$$\tau_{0N} = \frac{1}{\omega_0} \arccos \frac{q_1 \omega_0^4 + (p_1 q_2 - p_2 q_1)\omega_0^2 - p_3 q_2}{q_1^2 \omega_0^2 + q_2^2} + \frac{2N\pi}{\omega_0}, N = 0,1,2,\cdots$$

令 $\tau_0 = \tau_{00}$,现在证明横截条件

$$\left\{\frac{\mathrm{d}(\mathrm{Re}\lambda)}{\mathrm{d}\tau}\right\}_{\tau = \tau_0} > 0$$

成立.计算方程(16)对 τ 的微分,有

$$\left(\frac{\mathrm{d}\lambda}{\mathrm{d}\tau}\right)^{-1} = -\frac{3\lambda^2 + 2p_1\lambda + p_2}{\lambda(\lambda^3 + p_1\lambda^2 + p_1\lambda + p_0)} + \frac{q_1}{\lambda(q_1\lambda + q_2)} - \frac{\tau}{\lambda}.$$

计算可得

$$\mathrm{sgn}\left\{\frac{\mathrm{d}(\mathrm{Re}\lambda)}{\mathrm{d}\tau}\right\}_{\tau = \tau_0} = \mathrm{sgn}\left\{\mathrm{Re}\left(\frac{\mathrm{d}\lambda}{\mathrm{d}\tau}\right)^{-1}\right\}_{\lambda = \mathrm{i}\omega_0} =$$

$$\mathrm{sgn}\left\{-\frac{(p_2 - 3\omega_0^2)(\omega_0^2 - p_2) + 2p_1(p_3 - p_1\omega_0^2)}{(\omega_0^3 - p_2\omega_0)^2 + (p_3 - p_1\omega_0^2)^2} - \frac{q_1^2}{q_1^2\omega_0^2 + q_0^2}\right\}.$$

由(18)式可知

$$(\omega_0^3 - p_2\omega_0)^2 + (p_3 - p_1\omega_0^2)^2 = q_1^2\omega_0^2 + q_0^2.$$

有

$$\mathrm{sgn}\left\{\frac{\mathrm{d}(\mathrm{Re}\lambda)}{\mathrm{d}\tau}\right\}_{\tau = \tau_0} = \mathrm{sgn}\left\{\frac{3\omega_0^4 + 2(p_1^2 - 2p_2)\omega_0^2 + p_2^2 - 2p_1p_3 - q_1^2}{q_1^2\omega_0^2 + q_0^2}\right\} > 0.$$

因此,横截条件成立且当 $\omega = \omega_0, \tau = \tau_0$,系统(2)在 E^* 处存在 Hopf 分支.得到如下定理:

定理 4　(i)若 $p_3 > q_2$ 且(R1)~(R4)成立,则对于任意 $\tau \geq 0$,内部平衡点 E^* 是局部渐近稳定的.

(ii)若 $p_3 < q_2$ 且(R1)~(R4)成立,则存在一个 $\tau_0 > 0$,当 $\tau \in [0,\tau_0)$ 时,E^* 是局部渐近稳定的,且系统(2)在 $\tau = \tau_0$ 时经历 Hopf 分支,其中

$$\tau_0 = \frac{1}{\omega_0}\arccos \frac{q_1\omega_0^4 + (p_1q_2 - p_2q_1)\omega_0^2 - p_3q_2}{q_1^2\omega_0^2 + q_2^2}.$$

5　数值模拟

下面选定一系列参数值,利用 MATLAB 进行数值模拟,并给出相应的图形.选取 $r = 0.3, a = 1, b = 1,$ $c = 0.3, \alpha = 1, \beta = 0.8, n = 0.8, d_0 = 0.1, d_1 = 0.1, d_2 = 0.1.$

(1)若 $k_1 = 0.1, m = 0.05$,经计算可得 $\tau_0 = 22.77$.由定理 4 可知,当 $\tau = 23.77 > \tau_0$,内部平衡点 $E^* = (0.213\,10, 0.024\,68, 0.197\,40)$ 是不稳定的,系统(2)在 $\tau = \tau_0$ 时经历 Hopf 分支并产生周期解,见图 1.当 $\tau = 20 < \tau_0$ 时,内部平衡点 E^* 是局部渐近稳定的,见图 2.
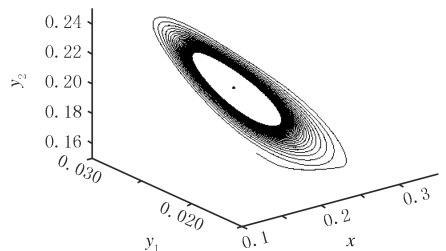
(2)若 $k = 0.1, m = 0.2, \tau = 20$,适当增加食饵避难的比例,由定理 4 可知,内部平衡点 $E^* = (0.257\,10, 0.026\,65, 0.213\,20)$ 仍局部渐近稳定,见图 3.

(3)若 $k = 0.5, m = 0.2, \tau = 20$,随着恐惧效应的增加,相较于图 3,系统的变化并不明显,此时内部平衡点 $E^* = (0.248\,20, 0.022\,28, 0.178\,30)$ 是局部渐近稳定的,见图 4.

(4)若 $k = 0.5, m = 0.95, \tau = 20$,系统(2)不存在内部平衡点 E^*,随着时间的增长,捕食者种群消亡.由定

图1　k=0.1,m=0.05, τ=23.77> τ_0时系统相图
Fig.1　Phase diagram of the system for k=0.1, m=0.05, τ=23.77> τ_0

理 3 可得,此时边界平衡点 $E_1 = (\frac{2}{3}, 0, 0)$ 是全局渐近稳定的,见图 5.

图2 k=0.1,m=0.05, τ=20< τ_0时系统相图
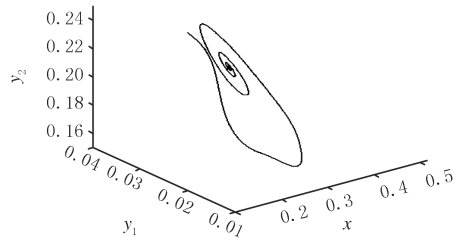Fig. 2 Phase diagram of the system for k=0.1, m=0.05, τ=20< τ_0

图3 k=0.1,m=0.2, τ=20时系统相图
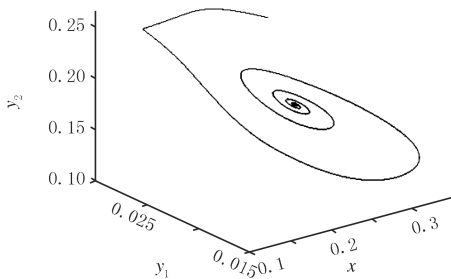Fig. 3 Phase diagram of the system for k=0.1, m=0.2, τ=20

图4 k=0.5,m=0.2, τ=20时系统相图
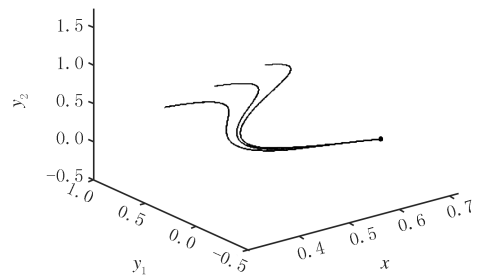Fig. 4 Phase diagram of the system for k=0.5, m=0.2, τ=20

图5 k=0.5,m=0.95, τ=20时系统相图
Fig. 5 Phase diagram of the system for k=0.5, m=0.95, τ=20

6 结 论

本文同时考虑了恐惧效应和食饵避难,研究了一类捕食者具有阶段性结构的时滞 Crowley-Martin 型捕食模型的平衡点性态.首先探讨了平衡点的存在性,发现食饵避难比例 m 控制着系统内部平衡点的存在性.当系统不存在唯一内部平衡点时,边界平衡点可以是全局渐近稳定的,恐惧效应 k 不影响边界平衡点的局部稳定性,却可以影响其全局稳定,而时滞并不会影响边界平衡点的稳定性.当系统只存在唯一内部平衡点时,该内部平衡点可以是局部渐近稳定的,但是时滞会使内部平衡点扰动,进而存在周期解.最后通过数值模拟,验证了文中的结论,还发现随着食饵避难和恐惧效应的增长,食饵数量趋于增长,捕食者数量最终趋于消亡,并且食饵避难对种群数量的影响远大于恐惧效应对种群数量的影响.

参 考 文 献

[1] KAR T K.Stability analysis of a prey-predator model incorporating a prey refuge[J].Communications in Nonlinear Science and Numerical Simulation,2005,10(6):681-691.

[2] HUANG Y,CHEN F,ZHONG L.Stability analysis of a prey-predator model with Holling type III response function incorporating a prey refuge[J].Applied Mathematics and Computation,2006,182(1):672-683.

[3] CHEN F,CHEN L,XIE X.On a Leslie-Gower predator-prey model incorporating a prey refuge[J].Nonlinear Analysis:Real World Applications,2009,10(5):2905-8.

[4] WEI F,FU Q.Hopf bifurcation and stability for predator-prey systems with Beddington-DeAngelis type functional response and stage structure for prey incorporating refuge[J].Applied Mathematical Modelling,2016,40(1):126-134.

[5] KHAJANCHI S,BANERJEE S.Role of constant prey refuge on stage structure predator-prey model with ratio dependent functional response[J].Applied Mathematics and Computation,2017,314:193-198.

[6] WANG X,ZANETTE L,ZOU X.Modelling the fear effect in predato-prey interactions[J].Journal of Mathematical Biology,2016,73(5):

1179-1204.

［7］ WANG X,ZOU X.Modeling the fear effect in predator-prey interactions with adaptive avoidance of predators[J].Bulletin of mathematical biology,2017,79(6):1325-1359.

［8］ 闫建博,刘贤宁.具有 Beddington-DeAngelis 功能反应及恐惧效应的捕食系统[J].西南大学学报(自然科学版),2018,282(6):115-120. YAN J B,LIU X N.A predator-prey system with Beddington-DeAngelis functional response and fear effect[J].Journal of Southwest University(Natural Science Edition),2018,282(6):115-120.

［9］ SHA A,SAMANTA S,MARTCHEVA M,et al.Backward bifurcation,oscillations and chaos in an eco-epidemiological model with fear effect[J].Journal of biological dynamics,2019,13(1):301-327.

［10］ ZHANG H,CAI Y,FU S,et al.Impact of the fear effect in a prey-predator model incorporating a prey refuge[J].Applied Mathematics and Computation,2019,356:328-337.

［11］ XIAO Z,LI Z,ZHU Z,et al.Hopf bifurcation and stability in a Beddington-DeAngelis predator-prey model with stage structure for predator and time delay incorporating prey refuge[J].Open Mathematics,2019,17(1):141-159.

［12］ MENG X,HUO H,XIANG H,et al.Stability in a predator-prey model with Crowley-Martin function and stage structure for prey[J].Applied mathematics and computation,2014,232:810-819.

［13］ TRIPATHI J P,TYAGI S,ABBAS S.Global analysis of a delayed density dependent predator-prey model with Crowley-Martin functional response[J].Communications in Nonlinear Science and Numerical Simulation,2016,30(1/2/3):45-69.

［14］ MAITI A P,DUBEY B,CHAKRABORTY A.Global analysis of a delayed stage structure prey-predator model with Crowley-Martin type functional response[J].Mathematics and Computers in Simulation,2019,162:58-84.

Stability of a delayed prey-predator model with fear effect and prey refuge

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Abstract：A delayed stage structure predator-prey model with Crowley-Martin type functional response incorporating prey refuge and fear effect was discussed. Firstly，the existence of equilibria was analyzed. Secondly，the conditions that the boundary equilibrium point satisfy the local and global asymptotically stability was obtained by discussing the roots of the characteristic equations and constructing Lyapunov function. Then，the influence of the time delay on the stability of the internal equilibrium point is studied，and the existence of the Hopf bifurcation at the internal equilibrium point of the system is analyzed. Finally，the results are verified by MATLAB numerical simulation.

Keywords：fear effect；prey refuge；time delay；stability；Hopf bifurcation

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