

# 常数比例投资下基于进入过程风险模型的渐近破产概率

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**摘 要:**研究了带投资的基于进入过程多险种的风险模型的破产概率, 其中允许保险公司将其资产按常数比例投资于满足几何布朗运动的股票市场, 其余部分投资于非负利率的债券市场, 假设索赔额分布属于  $D$  族且两两拟渐近独立, 根据伊藤公式, 给出保险公司资产的表达式, 最终得到了有限时间的破产概率.

**关键词:**保险风险模型; 两两拟渐近独立;  $D$  族; 几何布朗运动; 破产概率

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## 1 预备知识

随着金融市场的发展以及保险市场业务竞争日趋激烈, 单纯依靠保费收入使得保险公司的利润大幅减少, 甚至亏损. 投资利润已经成为现代保险公司利润中重要组成部分. 带投资的风险模型成为当今金融数学研究的热点问题之一.

当保险公司可以将其盈余投资于满足几何布朗运动的股票市场时, 一些研究者在经典的风险模型基础上作出了经典的工作. Browne<sup>[1]</sup>考虑满足漂移布朗运动的风险过程中关于指数效用和破产概率的最优策略. 在风险过程是复合泊松过程的假设下, Hipp 等<sup>[2]</sup>考虑了破产概率最小标准下的最优策略. 进一步, Caier 等<sup>[3]</sup>研究了保险公司将其盈余的常数比例部分投资在满足几何布朗运动的股票市场, 剩余资产投资于常利率的国债市场的问题, 用不同的方法得到了最终破产概率的相似结果. Chen<sup>[4]</sup>和 Yi<sup>[5]</sup>在前面研究的基础上, 对于索赔额两两拟渐近独立假设下得出了破产概率的渐近关系. 最近几年来, Chen<sup>[6]</sup>等又进一步得出了经典模型中常数比例投资下正则变化尾且相依索赔的渐近破产概率, Yang<sup>[7]</sup>等人得出了常利率投资下二维更新模型下的有限时间破产概率. 以上均是在经典风险模型下进行的研究. 本文突破经典模型, 研究了基于保单进入过程的多险种保险风险模型<sup>[8-11]</sup>, 假设索赔额满足  $D$  族且两两拟渐近独立时, 得出常数比例投资下的资产表达式, 并得出了有限时间破产概率的渐近表达式, 推广了相应结论, 以下为文中用到的定义.

**定义 1** 对于任一非负随机变量  $X$ , 如果矩母函数  $M_X(r) = E(e^{rX}) = \int_0^{\infty} e^{rx} dF(x) = \infty, \forall r > 0$ , 则称  $X$  (或称其分布函数  $F$ ) 属于重尾分布, 记为  $\mathcal{X}$ ; 相应地, 如果存在  $r > 0$  使得  $M_X(r) < \infty$ , 则称  $X$  (或称其分布  $F$ ) 属于轻尾分布, 记为  $\mathcal{X}^c$ .

下面给出几类重要的重尾分布族:

- 1) 称一个分布  $F(x)$  属于  $S$  族, 如果对任意的  $n \geq 2$  (或等价地  $n = 2$ ),  $F$  满足  $\lim_{x \rightarrow \infty} \frac{\overline{F^{*n}}(x)}{\overline{F}(x)} = n$ ;
- 2) 称一个分布  $F(x)$  属于  $\mathcal{D}$  族, 如果对任意固定的  $0 < y < 1$  (或等价地对  $y = \frac{1}{2}$ ),  $F$  满足

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$$\limsup_{x \rightarrow \infty} \frac{\bar{F}(xy)}{\bar{F}(x)} < \infty;$$

3) 称一个分布  $F(x)$  属于  $\mathcal{C}$  族, 分布  $F$  满足  $\lim_{x \downarrow 1} \liminf_{x \rightarrow \infty} \frac{\bar{F}(xy)}{\bar{F}(x)} = 1$ , 或等价地  $\lim_{x \uparrow 1} \limsup_{x \rightarrow \infty} \frac{\bar{F}(xy)}{\bar{F}(x)} = 1$ .

**定义 2** 称非负随机变量序列  $\{X_n, n \geq 1\}$  两两拟渐近独立, 如果对任意的  $X_i$  和  $X_j$ , 有

$$\lim_{x \rightarrow \infty} \frac{P(X_i > x, X_j > x)}{P(X_i > x) + P(X_j > x)} = 0.$$

**定义 3** 设  $X$  是定义在  $(-\infty, +\infty)$  的随机变量,  $F$  为其分布函数. 对任意  $v > 0$ , 记

$$\bar{F} * (v) = \liminf_{x \rightarrow \infty} \frac{\bar{F}(xv)}{\bar{F}(x)}, \bar{F}^* (v) = \limsup_{x \rightarrow \infty} \frac{\bar{F}(xv)}{\bar{F}(x)},$$

$$\text{以及 } J_F^+ = -\lim_{v \rightarrow \infty} \frac{\lg \bar{F} * (v)}{\lg v}, J_F^- = -\lim_{v \rightarrow \infty} \frac{\lg \bar{F}^* (v)}{\lg v}, L_F = \lim_{v \uparrow 1} \bar{F} * (v).$$

分别称  $J_F^+, J_F^-$  为  $F(x)$  的上、下 Matuszewska 指数. 易见  $0 \leq J_F^- \leq J_F^+ \leq \infty$ , 且有  $F \in D$ , 当且仅当  $J_F^+ < \infty$ .  $L_F$  也是  $F(x)$  的一个指数, 特别地, 当  $F \in C, L_F = 1$ .

**定义 4** 设  $f(\cdot), g(\cdot)$  都为正函数, (i) 如果  $\limsup_{x \rightarrow \infty} \frac{f}{g} \leq 1$ , 则  $f \preceq g$ ; (ii) 如果  $\liminf_{x \rightarrow \infty} \frac{f}{g} \geq 1$ , 则  $f \succeq g$ , 如果 (i) (ii) 都成立, 则  $f \sim g$ . 约定  $a \wedge b = \min\{a, b\}$ .

## 2 模型

$$U(t) = u + \sum_{i=1}^{N(t)} f(C_i) - \sum_{i=1}^{M(t)} X_i.$$

其中: (1)  $u(u \geq 0)$  是保险公司的初始资金;  $U(t)$  为盈余过程. (2)  $S_i$  是第  $i$  个保单的购买时刻, 满足  $0 < S_1 < S_2 < \dots$ , 记  $S_i$  生成的计数过程为进入过程  $N(t) = \sup\{i \geq 1, S_i \leq t\}, t \geq 0$ , 其均值函数为  $m(t)$ . (3) 假设保险公司提供  $k$  种产品供顾客选择, 第  $m$  种保单的保期为  $a_m, (m = 1, 2, \dots, k)$ , 第  $i$  个投保人选择一种保期为  $C_i$  的保单概率为  $P(C_i = a_k) = p_k, (k = 1, 2, \dots, m)$ , 其中  $f(\cdot)$  为单调非降的正值函数, 到时刻  $t$  保险公司的保费总收益  $\sum_{i=1}^{N(t)} f(C_i)$ . (4)  $X_i$  表示第  $i$  次的索赔额,  $T_i$  表示第  $i$  个投保人从购买保单时刻到发生索赔时刻之间的时间间隔, 假设它们独立同分布, 其分布为  $G(\cdot)$ . 至时刻  $t$  发生的索赔支付总额为  $\sum_{i=1}^{M(t)} X_i, M(t)$  表示至时刻  $t$  发生的索赔次数. (5)  $\{N(t), t \geq 0\}$  与  $\{f(C_i), i = 1, 2, \dots\}$  独立,  $\{M(t), t \geq 0\}$  是  $\{N(t), t \geq 0\}$  的随机选择.

以上是并不考虑盈余投资的模型. 现假设保险公司不仅投资于利率为  $r$  的国债市场, 而且投资于股票市场, 股票价格为过程  $S(t)$ , 由几何布朗运动定义如下:  $dS(t) = aS(t)dt + bS(t)dW(t)$ . 其中,  $a, b \in \mathbf{R}$  为常数,  $W(t)$  是标准布朗运动且独立于  $U(t)$ .

在很多国家, 保险公司将任一时刻盈余的常数比例投资于风险投资, 其余投资于债券市场. 如果在时刻  $t$ , 保险公式有资产  $Y(t)$ , 投资  $kY(t-)$  ( $0 \leq k \leq 1$ ) 在股票市场, 投资  $(1-k)Y(t-)$  在债券市场 (利息力为  $r$ ), 关于资产  $Y(t)$  的随机微分方程为  $dY(t) = dU(t) + kY(t-)(adt + bdW(t)) + (1-k)rY(t-)dt, t \geq 0$ .

根据伊藤公式, 得出资产  $Y(t)$  的表达式:

$$\begin{aligned} Y(t) &= e^{(\alpha - \frac{1}{2}\beta^2)t + \beta W(t)} \left[ u + \sum_{i=1}^{N(t)} f(C_i) e^{-(\alpha - \frac{1}{2}\beta^2)(S_i) - \beta W(S_i)} - \sum_{i=1}^{M(t)} X_i e^{-(\alpha - \frac{1}{2}\beta^2)(S_i + T_i) - \beta W(S_i + T_i)} \right] = \\ &= e^{\mu t + \beta W(t)} \left[ u + \sum_{i=1}^{N(t)} f(C_i) e^{-\mu(S_i) - \beta W(S_i)} - \sum_{i=1}^{M(t)} X_i e^{-\mu(S_i + T_i) - \beta W(S_i + T_i)} \right] = e^{\mu t + \beta W(t)} \left[ u + \right. \\ &\quad \left. \sum_{i=1}^{N(t)} f(C_i) e^{-\mu(S_i) - \beta W(S_i)} - \sum_{i=1}^{M(t)} X_i e^{-\mu(S_i + T_i) - \beta W(S_i + T_i)} 1_{\{S_i + T_i \leq t, T_i \leq C_i\}} \right]. \end{aligned} \quad (1)$$

这里  $\alpha := (a - r)k + r, \beta := bk, \mu := (\alpha - \frac{1}{2}\beta^2)$ .  $1_{\{S_i + T_i \leq t, T_i \leq C_i\}}$  表示  $M(t)$  对  $N(t)$  的随机选择.

跟往常一样,有限时间破产概率可以表示为:

$$\phi(u, t) = P(Y * (t) < 0, 0 < t \leq T, 0 < T < \infty), \quad (2)$$

终极破产概率表示为:

$$\phi(u) = P(Y(t) < 0, 0 < t < \infty). \quad (3)$$

### 3 主要结果及相关引理

下面是本文的主要结果.

**定理 1** 考虑模型(1),假设索赔额  $\{X_n, n \geq 1\}$  是两两拟渐近独立的非负随机变量序列,具有共同的分布函数  $F$ ,且  $F \in D, 0 < J_{\bar{F}} \leq J_F^+ < \infty$ ,则

$$\phi(u, T) \gtrsim L_F \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}(uy) dLN(y, \mu(s+h), \beta^2(s+h)) dG(h) dm(s),$$

$$\phi(u, T) \lesssim L_F^{-2} \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}(uy) dLN(y, \mu(s+h), \beta^2(s+h)) dG(h) dm(s),$$

其中  $\mu := (\alpha - \frac{1}{2}\beta^2)$ ,  $LN(y, a, b^2)$  定义为参数为  $a, b^2$  的对数正态分布. 特别地,若  $F \in \mathcal{C}$  族,则有下式成立:

$$\phi(u, T) \sim \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}(uy) dLN(y, \mu(s+h), \beta^2(s+h)) dG(h) dm(s).$$

在证明定理之前,我们需要以下引理,它们对本文主要结果的讨论是必要的.

**引理 1**<sup>[12]</sup>  $X$  是一个定义在  $[0, \infty)$  上的随机变量,  $F$  是它的分布函数,则

1)  $F \in D \Leftrightarrow L_F > 0$ ; 2)  $F \in D$ , 那么  $x^{-p} = o(\bar{F}F(x)), p > J_F^+$ ;  $\bar{F}F(x) = o(x^{-p}), p < J_F^-$ ; 3)  $F \in D$ , 对于  $p > J_F^+$  有  $EY^p < \infty$ , 那么

$$0 < E[\bar{F}F * (1/Y)] \leq \liminf_{t \rightarrow \infty} \frac{P(XY > t)}{\bar{F}(t)} \leq \limsup_{t \rightarrow \infty} \frac{P(XY > t)}{\bar{F}(t)} < E[\bar{F} * (1/Y)] < \infty.$$

**引理 2** 假设  $\{X_n, n \geq 1\}$  为非负两两拟渐近独立且同分布的随机变量序列,  $F$  为其共同的分布函数,  $F \in D, J_{\bar{F}} > 0$ , 且有如下其中之一关系成立:

1) 如果  $0 < J_F^+ < 1$ , 存在  $\lambda > 0$ , 使得  $J_{\bar{F}} - \lambda =: p_1 > 0, J_F^+ + \lambda =: p_2 < 1$ , 且有  $\sum_{i=1}^{\infty} E\Theta_i^{p_1} < \infty, \sum_{i=1}^{\infty} E\Theta_i^{p_2} < \infty$ ; 2) 如果  $J_F^+ \geq 1$ , 存在  $\lambda > 0$ , 使得  $J_{\bar{F}} =: p_1 > 0, J_F^+ + \lambda =: p_2$ , 且有  $\sum_{i=1}^{\infty} \{E\Theta_i^{p_1}\}^{\frac{1}{p_2}} < \infty, \sum_{i=1}^{\infty} \{E\Theta_i^{p_2}\}^{\frac{1}{p_2}} < \infty$ , 则  $\sum_{i=1}^{\infty} Pr(\Theta_i X_i > x) \lesssim Pr(\sum_{i=1}^{\infty} \Theta_i X_i > x) \lesssim L_F^{-2} \sum_{i=1}^{\infty} Pr(\Theta_i X_i > x)$ .

证明见 Yi<sup>[13]</sup>.

### 4 证明

令  $\tilde{Y}(t)$  为  $Y(t)$  的折现值, 即

$$\begin{aligned} \tilde{Y}(t) &:= Y(t)e^{-(\alpha - \frac{1}{2}\beta^2)t - \beta W(t)} = u + \sum_{i=1}^{N(t)} f(C_i)e^{-(\alpha - \frac{1}{2}\beta^2)S_i - \beta W(S_i)} - \\ &\sum_{i=1}^{M(t)} X_i e^{-(\alpha - \frac{1}{2}\beta^2)(S_i + T_i) - \beta W(S_i + T_i)} \mathbf{1}_{\{S_i + T_i \leq t, T_i \leq C_i\}} = u + \sum_{i=1}^{N(t)} f(C_i)e^{-(\alpha - \frac{1}{2}\beta^2)S_i - \beta W(S_i)} - \\ &\sum_{i=1}^{\infty} X_i e^{-(\alpha - \frac{1}{2}\beta^2)(S_i + T_i) - \beta W(S_i + T_i)} \mathbf{1}_{\{S_i + T_i \leq t, T_i \leq C_i\}} \mathbf{1}_{\{S_i \leq t\}}, t \geq 0. \end{aligned}$$

令  $\tilde{C}(t) := \sum_{i=1}^{N(t)} f(C_i)e^{-(\alpha - \frac{1}{2}\beta^2)S_i - \beta W(S_i)}$ ,  $\Theta_i := e^{-(\alpha - \frac{1}{2}\beta^2)(S_i + T_i) - \beta W(S_i + T_i)} \mathbf{1}_{\{S_i + T_i \leq t, T_i \leq C_i\}} \mathbf{1}_{\{S_i \geq t\}}, t \geq 0$ ,

$\tilde{X}X(t) := \sum_{i=1}^{\infty} X_i e^{-(\alpha - \frac{1}{2}\beta^2)(S_i + T_i) - \beta W(S_i + T_i)} \mathbf{1}_{\{S_i + T_i \leq t, T_i \leq C_i\}} \mathbf{1}_{\{S_i \leq t\}} = \sum_{i=1}^{\infty} X_i \Theta_i, t \geq 0$ , 则  $\tilde{Y}(t) = u + \tilde{C}(t) - \tilde{X}(t)$ , 其

中  $1_{\{S_i \leq t\}}, t \geq 0$  表示索赔是否发生.

(i) 若  $0 < J_F^+ < 1$ , 则对  $\forall 0 < \gamma < 1$ , 有

$$\begin{aligned} \sum_{i=1}^{\infty} E\Theta_i^\gamma &= \sum_{i=1}^{\infty} Ee^{-(\alpha-\frac{1}{2}\beta^2)(S_i+T_i)\gamma-\beta W(S_i+T_i)\gamma} 1_{\{S_i+T_i \leq t, T_i \leq C_i\}} 1_{\{S_i \leq t\}} = \\ &= \sum_{i=1}^{\infty} \int_0^T Ee^{-(\alpha-\frac{1}{2}\beta^2)(S_i+T_i)\gamma-\beta W(S_i+T_i)\gamma} 1_{\{S_i+T_i \leq t, T_i \leq C_i\}} dP(S_i \leq s) = \\ &= \sum_{i=1}^{\infty} \int_0^T \int_0^{((T-s) \wedge C_i)} Ee^{-(\alpha-\frac{1}{2}\beta^2)(s+h)\gamma-\beta W(s+h)} dP(T_i \leq h) dP(S_i \leq s) = \\ &= \sum_{k=1}^m p_k \int_0^T \int_0^{((T-s) \wedge a_k)} Ee^{-(\alpha-\frac{1}{2}\beta^2)(s+h)\gamma-\beta W(s+h)\gamma} dG(h) dm(s) = \\ &= \sum_{k=1}^m p_k \int_0^T \int_0^{((T-s) \wedge a_k)} e^{-(\alpha-\frac{1}{2}\beta^2)(s+h)\gamma+\frac{1}{2}\beta^2\gamma^2(s+h)} dG(h) dm(s) < \infty; \end{aligned}$$

(ii) 若  $1 \leq J_F^+ < \frac{2\alpha}{\beta^2} - 1$ , 选择  $\lambda > 0$ , 使得  $p_2 := J_F^+ + \gamma < \frac{2\alpha}{\beta^2} - 1$ , 令  $\xi_i = S_i - S_{i-1}, \eta_i = T_i - T_{i-1} - 1$ , 及  $S_0 = T_0 = 0$ , 从而  $\xi_i$  独立同分布,  $\eta_i$  独立同分布, 又因为布朗运动的独立性和平稳增量性, 对  $\forall 0 < v \leq p_2$ , 有

$$\begin{aligned} \sum_{i=1}^{\infty} (E\Theta_i^\gamma)^{\frac{1}{p_2}} &= \sum_{i=1}^{\infty} (Ee^{-(\alpha-\frac{1}{2}\beta^2)(S_i+T_i)v-\beta W(S_i+T_i)v} 1_{\{S_i+T_i \leq t, T_i \leq C_i\}} 1_{\{S_i \leq t\}})^{\frac{1}{p_2}} \leq \sum_{i=1}^{\infty} (Ee^{-(\alpha-\frac{1}{2}\beta^2)(S_i+T_i)v-\beta W(S_i+T_i)v})^{\frac{1}{p_2}} = \\ &= \sum_{i=1}^{\infty} \left[ E \left( e^{-(\alpha-\frac{1}{2}\beta^2)v(S_i-S_{i-1})+(T_i-T_{i-1})v-\beta W[(S_i-S_{i-1})+(T_i-T_{i-1})]v} \cdot e^{-(\alpha-\frac{1}{2}\beta^2)v(S_{i-1}-S_{i-2})+(T_{i-1}-T_{i-2})v-\beta W[(S_{i-1}-S_{i-2})+(T_{i-1}-T_{i-2})]} \dots \right. \right. \\ &\quad \left. \left. e^{-(\alpha-\frac{1}{2}\beta^2)v(S_1-S_0)+(T_1-T_0)v-\beta W[(S_1-S_0)+(T_1-T_0)]} \right) \right]^{\frac{1}{p_2}} = \sum_{i=1}^{\infty} \left[ E \left( e^{-(\alpha-\frac{1}{2}\beta^2)v(\xi_i+\eta_i)-\beta W(\xi_i+\eta_i)} \cdot \right. \right. \\ &\quad \left. \left. e^{-(\alpha-\frac{1}{2}\beta^2)v(\xi_{i-1}+\eta_{i-1})-\beta W(\xi_{i-1}+\eta_{i-1})} \dots \cdot e^{-(\alpha-\frac{1}{2}\beta^2)v(\xi_1+\eta_1)-\beta W(\xi_1+\eta_1)} \right) \right]^{\frac{1}{p_2}} = \\ &= \sum_{i=1}^{\infty} \left[ \prod_{k=1}^i Ee^{-(\alpha-\frac{1}{2}\beta^2)v(\xi_k+\eta_k)-\beta W(\xi_k+\eta_k)} \right]^{\frac{1}{p_2}} = \sum_{i=1}^{\infty} (Ee^{-(\alpha-\frac{1}{2}\beta^2)v(\xi_1+\eta_1)-\beta W(\xi_1+\eta_1)})^{\frac{i}{p_2}} < \infty, \end{aligned}$$

最后上式成立是因为  $0 < Ee^{-(\alpha-\frac{1}{2}\beta^2)v(\xi_k+\eta_k)-\beta W(\xi_k+\eta_k)} = Ee^{-(\alpha-\frac{1}{2}\beta^2)v(\xi_1+\eta_1)+\frac{1}{2}v^2\beta^2(\xi_1+\eta_1)} < 1$  因此,  $\Theta_i$  满足引理 2 的条件.

由上述, 可将有限时间破产概率记为

$$\psi(u, T) = P(\tilde{Y}(t) < 0, 0 < t < T), u - \tilde{X}(T) \leq \tilde{Y}(t) \leq u + \tilde{C}(T) - \tilde{X}(t). \tag{5}$$

根据(5)式以及引理 2, 得

$$\begin{aligned} \psi(u, T) &\leq P(\tilde{X}(T) > u) = P\left(\sum_{i=1}^{\infty} X_i e^{-(\alpha-\frac{1}{2}\beta^2)(S_i+T_i)-\beta W(S_i+T_i)} 1_{\{S_i+T_i \leq T, T_i \leq C_i\}} 1_{\{S_i \leq T\}} > u\right) \approx \\ &= L_F^{-2} \sum_{i=1}^{\infty} P(X_i e^{-(\alpha-\frac{1}{2}\beta^2)(S_i+T_i)-\beta W(S_i+T_i)} 1_{\{S_i+T_i \leq T, T_i \leq C_i\}} 1_{\{S_i \leq T\}} > u) = \\ &= L_F^{-2} \sum_{i=1}^{\infty} \int_0^T P(X_i e^{-(\alpha-\frac{1}{2}\beta^2)(S_i+T_i)-\beta W(S_i+T_i)} 1_{\{s+T_i \leq T, T_i \leq C_i\}} > u) dP(S_i < s) = \\ &= L_F^{-2} \sum_{i=1}^{\infty} \int_0^T P(X_i e^{-(\alpha-\frac{1}{2}\beta^2)(S_i+T_i)-\beta W(S_i+T_i)} 1_{\{T_i \leq T-s, T_i \leq C_i\}} > u) dP(S_i < s) = \\ &= L_F^{-2} \sum_{i=1}^{\infty} \int_0^T \int_0^{(T-s) \wedge C_i} P(X_i e^{-(\alpha-\frac{1}{2}\beta^2)(s+h)-\beta W(s+h)} > u) dP(T_i \leq h) dP(S_i \leq s) = \\ &= L_F^{-2} \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} P(X_i e^{-(\alpha-\frac{1}{2}\beta^2)(s+h)-\beta W(s+h)} > u) dG(h) dm(s) = \\ &= L_F^{-2} \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^{\infty} \bar{F}(uy) dLN(y, (\alpha - \frac{1}{2}\beta^2)(s+h), \beta^2(s+h)) dG(h) dm(s), \tag{6} \end{aligned}$$

即得

$$\phi(u, T) \leq P(\tilde{X}(T) > u) = L_F^{-2} \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}(uy) dLN(y, \mu(s+h), \beta^2(s+h)) dG(h) dm(s).$$

从而得到了有限时间破产概率的上界. 现在考虑其下界, 由(5)式, 对  $\forall 0 < \epsilon < 1$ , 有  $\phi(u, T) \geq P(\tilde{X}(t) > u + \tilde{C}(T), 0 \leq t \leq T) = P(\tilde{X}(T) > u + \tilde{C}(T)) \geq P(\tilde{X}(T) > (1 + \epsilon)u) - P(\tilde{C}(T) > \epsilon u)$ .

由引理 2 可得

$$\begin{aligned} P(\tilde{X}(T) > (1 + \epsilon)u) &= P\left(\sum_{i=1}^\infty X_i e^{-(\alpha - \frac{1}{2}\beta^2)(S_i + T_i) - \beta W(S_i + T_i)} 1_{\{S_i + T_i \leq T, T_i \leq C_i\}} 1_{\{S_i \leq T\}} > (1 + \epsilon)u\right) \geq \\ &\sum_{i=1}^\infty P\left(X_i e^{-(\alpha - \frac{1}{2}\beta^2)(S_i + T_i) - \beta W(S_i + T_i)} 1_{\{S_i + T_i \leq T, T_i \leq C_i\}} 1_{\{S_i \leq T\}} > (1 + \epsilon)u\right) = \\ &\sum_{i=1}^\infty \int_0^T P\left(X_i e^{-(\alpha - \frac{1}{2}\beta^2)(S_i + T_i) - \beta W(S_i + T_i)} 1_{\{S_i + T_i \leq T, T_i \leq C_i\}} > (1 + \epsilon)u\right) dP(S_i \leq s) = \\ &\sum_{i=1}^\infty \int_0^T \int_0^{(T-s) \wedge C_i} P\left(X_i e^{-(\alpha - \frac{1}{2}\beta^2)(s+h)} > (1 + \epsilon)u\right) dP(T_i \leq h) dP(S_i \leq s) = \\ &\sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} P\left(X_i e^{-(\alpha - \frac{1}{2}\beta^2)(s+h) - \beta W(s+h)} > (1 + \epsilon)u\right) dG(h) dm(s) = \\ &\sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}((1 + \epsilon)uy) dLN(y, (\alpha - \frac{1}{2}\beta^2)(s+h), \beta^2(s+h)) dG(h) dm(s) \geq \\ &\bar{F}_*(1 + \epsilon) \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}(uy) dLN(y, (\alpha - \frac{1}{2}\beta^2)(s+h), \beta^2(s+h)) dG(h) dm(s). \end{aligned} \tag{7}$$

令  $\epsilon \rightarrow 0^+, u := (\alpha - \frac{1}{2}\beta^2)$ , 结合(7)式, 有

$$\begin{aligned} \lim_{\substack{u \rightarrow \infty \\ \downarrow 0}} P(\tilde{X}(T) > (1 + \epsilon)u) &\geq \lim_{\substack{u \rightarrow \infty \\ \downarrow 0}} \bar{F}_*(1 + \epsilon) \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}(uy) dLN(y, \mu(s+h), \\ &\beta^2(s+h)) dG(h) dm(s) \geq L_F \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}(uy) dLN(y, \mu(s+h), \beta^2(s+h)) dG(h) dm(s), \end{aligned}$$

可知  $\tilde{C}(T)$  有任意阶有限矩, 则  $\lim_{x \rightarrow \infty} x^\rho(\tilde{C}(T) > x) = 0, \rho > 0$ , 又由引理 1 知  $2(\alpha - \frac{1}{2}\beta^2)/\beta^2 > \rho > J_F^+$  时,

$$\lim_{x \rightarrow \infty} x^\rho \bar{F}(x) = +\infty \text{ 所以有 } \lim_{x \rightarrow \infty} \frac{P(\tilde{C}(T) > x)}{\bar{F}(x)} = \lim_{x \rightarrow \infty} \frac{x^\rho P(\tilde{C}(T) > x)}{x^\rho \bar{F}(x)} = 0, \rho > J_F^+.$$

再结合 Fatou 引理,  $F \in D$ , 以及引理 1 的(3), 可知对于任意的  $\epsilon > 0$ ,

$$\begin{aligned} &\limsup_{u \rightarrow \infty} \frac{P(\tilde{C}(T) > \epsilon u)}{\sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}(uy) dLN(y, \mu(s+h), \beta^2(s+h)) dG(h) dm(s)} = \\ \limsup_{u \rightarrow \infty} \frac{P(\tilde{C}(T) > \epsilon u)}{\bar{F}(\epsilon u)} \frac{\bar{F}(\epsilon u)}{\bar{F}(u)} &\left\{ \frac{\sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}(uy) dLN(y, \mu(s+h), \beta^2(s+h)) dG(h) dm(s)}{\bar{F}(u)} \right\}^{-1} = \\ \limsup_{u \rightarrow \infty} \frac{P(\tilde{C}(T) > \epsilon u)}{\bar{F}(\epsilon u)} \limsup_{u \rightarrow \infty} \frac{\bar{F}(\epsilon u)}{\bar{F}(u)} &\left\{ \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \frac{\bar{F}(uy) dLN(y, \mu(s+h), \beta^2(s+h))}{\bar{F}(u)} dG(h) dm(s) \right\}^{-1} \leq \\ \limsup_{u \rightarrow \infty} \frac{P(\tilde{C}(T) > \epsilon u)}{\bar{F}(\epsilon u)} \limsup_{u \rightarrow \infty} \frac{\bar{F}(\epsilon u)}{\bar{F}(u)} &\left\{ \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}_*(uy) \cdot \right. \\ &\left. dLN(y, \mu(s+h), \beta^2(s+h)) dG(h) dm(s) \right\}^{-1} = 0. \end{aligned}$$

即得

$$\phi(u, T) \geq L_F \sum_{k=1}^m p_k \int_0^T \int_0^{(T-s) \wedge a_k} \int_0^\infty \bar{F}(uy) dLN(y, \mu(s+h), \beta^2(s+h)) dG(h) dm(s). \tag{8}$$

结合(6)、(8)式, 定理得证. 特别地, 当索赔额分布  $F \in C$  时, 结论显然成立.

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## Asymptotic Ruin Probabilities for Proportional Investment Under Interest Force of a Risk Model Based on Entrance Process with Dominatedly-Varying-Tailed Claims

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**Abstract:** The asymptotic behavior of ruin probabilities was investigated in a risk model based on the policy entrance process, in which the insurance company is allowed to invest a constant fraction of its wealth in a stock market which is described by a geometric Brownian motion and the remaining wealth in a bond with nonnegative interest force. For this model, in the presence of pairwise quasi-asymptotically independent and dominant varying tailed claims, the expression of the wealth process was derived by the its formula, and the finite-time ruin probabilities were obtained.

**Keywords:** small insurance risk model; pairwise quasi-asymptotical independence; class  $D$ ; ruin probability