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具有时滞的递归神经网络模型的分支分析

刘 霞,焦建锋

(河南师范大学 数学与信息科学学院;大数据统计分析与优化控制河南省工程实验室,河南 新乡 453007)

摘 要:主要研究了一类含有 6 个时滞的四阶神经网络模型的分支问题. 通过应用时滞微分方程的中心流形定理和规范型理论,得到了系统在原点处的 Bogdanov-Takens (B-T) 分支和 triple zero 分支的规范型,进而给出了一些主要的分支现象.

关键词:神经网络模型;B-T 分支;triplezero 分支;规范型

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1 分支的存在性

近几年,各个领域的学者和科研工作者都对神经网络模型十分感兴趣[1-5],在文献[5]中主要研究了含 3 个神经元神经网络模型的 Hopf 分支,共振和非共振的双 Hopf 分支,受该文的启发,本文主要考虑如下的含有 4 个神经元神经网络模型的 B-T 分支和 triple zero 分支问题.

$$\begin{cases}\dot{x}_1 = -x_1(t)+f(x_2(t-\tau_2)),\dot{x}_2 = -x_2(t)+f(x_3(t-\tau_4)),\dot{x}_3 = -x_3(t)+f(x_4(t-\tau_6)),\\ \dot{x}_4 = -x_4(t)+af(x_1(t-\tau_1))+b\,f(x_2(t-\tau_3))+c\,f(x_3(t-\tau_5)),\end{cases} \tag{1}$$

其中 $x_i(i=1,2,3,4)$ 表示第 i 个神经元的状态,a,b,c 表示连接权,τ_i 是非负时滞,$f(x)$ 是神经元间诱发的非线性函数,且满足 $f(0)=0,\ f'(0)=1$.

令 $u_1(t)=x_1(t),u_2(t)=x_2(t-\tau_2),u_3(t)=x_3(t-\tau_2-\tau_4),u_4(t)=x_4(t-\tau_2-\tau_4-\tau_6)$,则系统(1)可以写成如下形式

$$\begin{cases}\dot{u}_1 = -u_1(t)+f(u_2(t)),\dot{u}_2 = -u_2(t)+f(u_3(t)),\dot{u}_3 = -u_3(t)+f(u_4(t)),\\ \dot{u}_4 = -u_4(t)+af(u_1(t-\sigma_1))+bf(u_2(t-\sigma_2))+c\,f(u_3(t-\sigma_3)),\end{cases} \tag{2}$$

其中 $\sigma_1=\tau_1+\tau_2+\tau_4+\tau_6,\sigma_2=\tau_3+\tau_4+\tau_6,\sigma_3=\tau_5+\tau_6$.

不失一般性,假设 $\tau_1>\tau_2>\tau_3>\tau_4>\tau_5>\tau_6>0$,可得 $\sigma_1>\sigma_2>\sigma_3$. 系统(2)在原点处的特征方程是

$$F(\lambda)=(\lambda+1)^4-ae^{-\lambda\sigma_1}-(\lambda+1)be^{-\lambda\sigma_2}-(\lambda+1)^2ce^{-\lambda\sigma_3}=0. \tag{3}$$

由方程(3)可以得到

$$F(0)=1-a-b-c,F'(0)=4-b-2c+a\sigma_1+b\sigma_2+c\sigma_3,$$
$$F''(0)=12-2c-a\sigma_1^2+2b\sigma_2-b\sigma_2^2+4c\sigma_3-c\sigma_3^2. \tag{4}$$

从 $F(0)=F'(0)=0$ 解得

$$a=a_0=\frac{(c-1)\sigma_2-c\sigma_3+c-3}{\sigma_1-\sigma_2+1},b=b_0=-\frac{(c-1)\sigma_1-c\sigma_3+2c-4}{\sigma_1-\sigma_2+1}. \tag{5}$$

通过 $F''(0)=0$,得出

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第 1 作者简介(通信作者):刘 霞(1980—),女,河南周口人,河南师范大学副教授,研究方向为微分方程分支理论,E-mail:liuxiapost@163.com.

$$c=c_0=\frac{(\sigma_2+3)\sigma_1^2+(12-\sigma_2^2+2\sigma_2)\sigma_1+12-4\sigma_2^2-4\sigma_2}{(\sigma_2-\sigma_3+1)\sigma_1^2+m_0\sigma_1+(\sigma_3-2)\sigma_1^2+(2-\sigma_3^2+2\sigma_3)\ \sigma_2+\sigma_3^2-4\sigma_3+2},$$

其中 $m_0=-\sigma_2^2+2\sigma_2+\sigma_3^2-4\sigma_3+2$.

因此,可以计算出 $F'''(0)\neq 24+a\sigma_1^3-3b\sigma_2^2+b\sigma_2^3+6c\sigma_3-6c\sigma_3^2+c\sigma_3^3$,假设(H0)$F'''(0)\neq 0$ 成立.因此,可总结如下引理.

引理1 i) 如果(H1)$a=a_0,b=b_0,c\neq c_0$ 成立,则 $\lambda=0$ 是方程(3) 的二重零根;(ii) 如果(H0) 和(H2) $a=a_0,b=b_0,c=c_0$ 成立,则 $\lambda=0$ 是方程(3) 的三重零根.

2 B-T 分支的规范型

为了讨论系统(2) 在原点处的 B-T 分支,把 a_0 和 b_0 当作分支参数.因此,令 $a=a_0+\mu_1$, $b=b_0+\mu_2$,得到下面的系统:

$$\begin{cases}\dot{u}_1=-u_1(t)+f(u_2(t)),\dot{u}_2=-u_2(t)+f(u_3(t)),\dot{u}_3=-u_3(t)+f(u_4(t)),\\ \dot{u}_4=-u_4(t)+(a_0+\mu_1)f(u_1(t-\sigma_1))+(b_0+\mu_2)f(u_2(t-\sigma_2))+cf(u_3(t-\sigma_3)),\end{cases}\tag{6}$$

其中 $\mu=(\mu_1,\mu_2)$ 在原点(0,0) 附近充分小.令 $k=f''(0)$,系统(6) 的泰勒展开式如下:

$$\dot{U}(t)=AU(t)+\sum_{i=1}^{3}B_iU(t-\sigma_i)+\frac{1}{2}\hat{F}_2(U_t,P)+\cdots,\tag{7}$$

为了获得系统(6) 在原点处的规范型,需要计算系统(7) 的广义特征空间 P 的基 $\Phi(\theta)$ 及其对偶空间 P^* 的基 $\Psi(s)$,其中 $\Phi(\theta)$ 和 $\Psi(s)$ 的选取详见文献[6].通过计算可得

$$\Phi(\theta)=\begin{pmatrix}1&\theta\\1&\theta+1\\1&\theta+2\\1&\theta+3\end{pmatrix},\Psi(0)=\begin{pmatrix}v_{11}&v_{12}&v_{13}&v_{14}\\v_{21}&v_{22}&v_{23}&v_{24}\end{pmatrix},\tag{8}$$

且 $\dot{\Phi}=\Psi J_0,\Psi=-J_0\Psi$ 和 $\langle\Phi,\Psi\rangle=I$ 成立,其中 $J_0=\begin{pmatrix}0&1\\0&0\end{pmatrix}$,

$$v_{11}=\frac{2n_0(c\sigma_2-c\sigma_3+c-\sigma_2-3)}{3l^2},v_{13}=\frac{(\sigma_1-\sigma_2+1)\ [v_{11}+(\sigma_1+2)v_{21}]}{c\sigma_2-c\sigma_3+c-\sigma_2-3},$$

$$v_{12}=\frac{(\sigma_1-\sigma_2+1)[(c-1)\ v_{11}+(c\sigma_1-c\sigma_3+c-\sigma_1-3)v_{21}]}{-c\sigma_2+c\sigma_3-c+\sigma_2+3},$$

$$v_{14}=\frac{(-\sigma_1+\sigma_2-1)[v_{11}+(\sigma_1+1)v_{21}]}{-c\sigma_2+c\sigma_3-c+\sigma_2+3},v_{21}=-\frac{2(c\sigma_2-c\sigma_3+c-\sigma_2-3)}{l},$$

$$v_{22}=\frac{2(\sigma_1-\sigma_2+1)(c-1)}{l},v_{23}=v_{24}=-\frac{2(\sigma_1-\sigma_2+1)}{l},$$

$$l=(c\sigma_2-c\sigma_3+c-\sigma_2-3)\sigma_1^2+[(1-c)\sigma_2^2+(2c-2)\sigma_2+c\sigma_3^2-4c\sigma_3+2c-12]\sigma_1+(c\sigma_3-2c+4)\sigma_2^2+(-c\sigma_3^2+2c\sigma_3+2c+4)\sigma_2+c\sigma_3^2-4c\sigma_3+2c-12,$$

$$n_0=\{(2\sigma_2-2\sigma_3+2)\sigma_1^3+(-3\sigma_2^2+3\sigma_3^2+9\sigma_2-15\sigma_3+9)\sigma_1^2+[\sigma_2^3+(3\sigma_3-12)\sigma_2^2+(-3\sigma_3^2+6\sigma_3+12)\ \sigma_2-\sigma_3^3+\sigma_3^2-\sigma_3+12]\sigma_1+(2-\sigma_3)\sigma_2^3+6(\sigma_3-2)\sigma_2^2+(\sigma_3^3-9\sigma_3^2+12\sigma_3+6)\sigma_2-\sigma_3^3+9\sigma_3^2-18\sigma_3+6\}c-2(\sigma_2+3)\sigma_1^3+(3\sigma_2^2-9\sigma_2-45)\sigma_1^2+(-\sigma_2^3+18\sigma_2^2+6\sigma_2-96)\sigma_1-4\sigma_2^3+24\sigma_2^2+36\sigma_2-60.$$

令 $z=\begin{pmatrix}z_1\\z_2\end{pmatrix}$.根据文献[7-8],系统(6) 可被分解成

$$\dot{z}=J_0z+\frac{1}{2!}f_2^1(z,y,\mu)+\frac{1}{3!}f_3^1(z,y,\mu)+\cdots,$$

$$\dot{y}=A_0|_Q+\frac{1}{2!}f_2^2(z,y,\mu)+\frac{1}{3!}f_3^2(z,y,\mu)+\cdots.\tag{9}$$

其中 $f_j^1=\Psi(0)F(\Phi x+y,\mu),f_j^2=(I-\pi)X_0F(\Phi x+y,\mu),j=1,2$.然后,根据文献[7],通过应用空间

$\mathrm{Im}M_2^1$ 和 $(\mathrm{Im}M_2^1)^c$ 的基,定义 $P_{I,2}p = a$ 满足 $p - a \in (\mathrm{Im}M_2^1)^c$. 对任意的 $p \in V_2^4\begin{pmatrix}R_2\\0\end{pmatrix}$,知 $\mathrm{Proj}(\mathrm{Im}M_2^1)^c(p)$ 有以下几种情况:

$$\mathrm{Proj}_{(\mathrm{Im}M_2^1)^c}(p) = \begin{cases} p, p \in \mathrm{Im}(M_2^1)^c, \\ 0, p \in \mathrm{Im}(M_2^1), \end{cases} \quad \mathrm{Proj}_{(\mathrm{Im}M_2^1)^c}\begin{pmatrix}0\\2z_1z_2\end{pmatrix} = \begin{pmatrix}0\\2z_1z_2\end{pmatrix},$$

$$\mathrm{Proj}_{(\mathrm{Im}M_2^1)^c}\begin{pmatrix}z_1\mu_1\\0\end{pmatrix} = \begin{pmatrix}0\\z_2\mu_1\end{pmatrix}, \mathrm{Proj}_{(\mathrm{Im}M_2^1)^c}\begin{pmatrix}z_1\mu_2\\0\end{pmatrix} = \begin{pmatrix}0\\z_2\mu_2\end{pmatrix}.$$

因此,系统在 B-T 奇点处的规范型有如下的形式:

$$\dot{z} = J_0 z + \frac{1}{2}g_2^1(z,0,\mu) + \frac{1}{3}g_3^1(z,0,\mu) + \cdots, \tag{10}$$

其中 $g_j^1(z,0,\mu)$ 是在 (z,μ) 上的 $j(j \geq 2)$ 次齐次多项式,且 $g_j^1 = (I - P_I, j^1)\bar{f}_j^1 (j \geq 2)$ 参考文献([7,9]). 由 $\varphi = \Phi(\theta)z$ 和(8) 可得出

$$\varphi_1(0) = z_1 + z_2, \varphi_1(-\sigma_1) = z_1 + (1-\sigma_1)z_2, \varphi_2(0) = z_1 + 2z_2,$$
$$\varphi_2(-\sigma_2) = z_1 + (2-\sigma_2)z_2, \varphi_3(0) = z_1 + 3z_2, \varphi_3(-\sigma_3) = z_1 + (3-\sigma_3)z_2, \varphi_4(0) = z_1 + 4z_2. \tag{11}$$

然后,由(8),(9) 和(11),得到

$$\frac{1}{2}f_2^1(\tilde{x},0,\mu) = \Psi(0)\left[\frac{1}{2}\hat{F}_2(\Phi\tilde{x},\mu)\right] = \begin{pmatrix} v_{11} & v_{12} & v_{13} & v_{14} \\ v_{21} & v_{22} & v_{23} & v_{24} \end{pmatrix} \times$$
$$\begin{pmatrix} \frac{k}{2}\varphi_2^2(0) \\ \frac{k}{2}\varphi_3^2(0) \\ \frac{k}{2}\varphi_4^2(0) \\ \sum_{i=1}^{2}\mu_i\varphi_i(-\sigma_i) + \frac{a_0k}{2}\varphi_1^2(-\sigma_1) + \frac{b_0k}{2}\varphi_2^2(-\sigma_2) + \frac{ck}{2}\varphi_3^2(-\sigma_3) \end{pmatrix} =$$
$$\begin{pmatrix} a_{2000}z_1^2 + a_{1100}z_1z_2 + v_{14}z_1\mu_1 + v_{14}z_1\mu_2 + a_{0200}z_2^2 + \\ v_{14}(\sigma_1+1)z_2\mu_1 + v_{14}(\sigma_2+2)z_2\mu_2 b_{2000}z_1^2 + \\ b_{1100}z_1z_2 + v_{24}z_1\mu_1 + v_{24}z_1\mu_2 + b_{0200}z_2^2 + \\ v_{24}(\sigma_1+1)z_2\mu_1 + v_{24}(\sigma_2+2)z_2\mu_2 \end{pmatrix}, \tag{12}$$

其中

$$a_{2000} = \frac{k}{2}(v_{11} + v_{12} + v_{13} + v_{14}),\ a_{0200} = \frac{k}{2}(4v_{11} + 9\ v_{12} + 16v_{13}) + \frac{ke_1v_{14}}{2(\sigma_1 + 1 - \sigma_2)},$$

$$a_{1100} = (2\ v_{11} + 3\ v_{12} + 4\ v_{13})\ k + \frac{k(2\ c\sigma_1 - 4\ c\sigma_2 + 2\ c\sigma_3 - \sigma_1 + 3\ \sigma_2 + 5)v_{14}}{\sigma_1 + 1 - \sigma_2},$$

$$b_{2000} = \frac{k}{2}(v_{21} + v_{22} + v_{23} + v_{24}),\ b_{0200} = \frac{k}{2}(4\ v_{21} + 9\ v_{22} + 16\ v_{23}) + \frac{ke_1v_{24}}{2(\sigma_1 + 1 - \sigma_2)},$$

$$b_{1100} = (2\ v_{21} + 3\ v_{22} + 4\ v_{23})k + \frac{k(2\ c\sigma_1 - 4\ c\sigma_2 + 2\ c\sigma_3 - \sigma_1 + 3\ \sigma_2 + 5)v_{24}}{\sigma_1 + 1 - \sigma_2},$$

$$e_1 = [(\sigma_2 - \sigma_3)(\sigma_1 - \sigma_3)(\sigma_1 - \sigma_2) + \sigma_1^2 - 2\sigma_1\sigma_2 + 4\sigma_3\sigma_1 - 2\sigma_2^2 - 2\sigma_2\sigma_3 + \sigma_3^2 + 7\sigma_1 -$$
$$16\sigma_2 + 9\sigma_3 + 2]c - \sigma_1^2\sigma_2 + \sigma_1\sigma_2^2 - 3\sigma_1^2 + 2\sigma_1\sigma_2 + 4\sigma_2^2 - 2\sigma_1 + 15\sigma_2 + 13.$$

因此,从系统(10),可以得到

$$\dot{z}_1 = z_2, \dot{z}_2 = \lambda_1 z_1 + \lambda_2 z_2 + \gamma_1 z_1^2 + \gamma_2 z_1 z_2 + \cdots, \tag{13}$$

其中

$$\lambda_1 = v_{24}(\mu_1 + \mu_2), \lambda_2 = [v_{14} + v_{24}(\sigma_1 + 1)]\mu_1 + [v_{14} + v_{24}(\sigma_2 + 2)]\mu_2, \gamma_1 = b_{2000},$$

$$\gamma_2 = 2a_{2000} + b_{1100} = \frac{2k[sol - (c\sigma_1 - 2c\sigma_2 + c\sigma_3 - 3\sigma_1 + 4\sigma_2)n_0]}{3l^2},$$

$$s_0 = 3(\sigma_1 + 1 - \sigma_2)(c\sigma_1 - c\sigma_3 + 2c - 3\sigma_1 - 12).$$

进一步，易得到 $\left|\frac{\partial(\lambda_1,\lambda_2)}{\partial(\mu_1,\mu_2)}\right| = v_{24}^2(\sigma_1 - \sigma_2 + 1) > 0.$

因此，映射 $(\mu_1, \mu_2) \to (\lambda_1,\lambda_2)$ 是正则的. 可以得出如下的定理.

定理 1 如果 $k = f''(0) \neq 0$，系统(7) 在中心流形上等价于系统(13).

参考文献[6]，(13) 式有两个平衡点 $E_1 = (0,0)$ 和 $E_2 = (-\frac{\lambda_1}{\gamma_1},0)$，在原点附近以 μ_1 和 μ_2 为分支参数的分支曲线分析如下：

1) (13) 式在曲线 $S = \{(\lambda_1,\lambda_2): \lambda_1 = 0, \lambda_2 \in R\} = \{(\mu_1,\mu_2): v_2 4(\mu_1 + \mu_2) = 0\}$ 经历一个超临界分支；

2) 在平凡平衡点 E_1 处，(13) 式在半曲线 $H = \{(\lambda_1,\lambda_2): \lambda_2 = 0, \lambda_1 < 0\} = \{(\mu_1,\mu_2): [v_1 4 + v_{24}(\sigma_1 + 1)]\mu_1 + [v_{14} + v_{24}(\sigma_2 + 2)]\mu_2 = 0, v_{24}(\mu_1 + \mu_2) < 0\}$ 上经历了一个不稳定的 Hopf 分支；

3) 在平衡点 E_2 处，(13) 式在半曲线 $T = \{(\lambda_1,\lambda_2): \lambda_2 = \frac{\gamma_2}{\gamma_1}\lambda_1, \lambda_1 > 0\} = \{(\mu_1,\mu_2): [v_{14} + v_{24}(\sigma_1 + 1)]\mu_1 + [v_{14} + v_{24}(\sigma_2 + 2)]\mu_2 = \frac{\gamma_2}{\gamma_1}v_{24}(\mu_1 + \mu_2), v_{24}(\mu_1 + \mu_2) > 0\}$ 上经历了一个稳定的 Hopf 分支.

3 Triple zero 分支的规范型

在这一部分，把 a_0, b_0, c_0 看作分支参数来讨论系统(2) 在原点处的三重分支. 令 $a = a_0 + \beta_1$，$= b_0 + \beta_2$，$c = c_0 + \beta_3$，由此，得到如下的系统：

$$\dot{u}_1 = - u_1(t) + f(u_2(t)), \dot{u}_2 = - u_2(t) + f(u_3(t)), \dot{u}_3 = - u_3(t) + f(u_4(t)),$$
$$\dot{u}_4 = - u_4(t) + (a_0 + \beta_1)f(u_1(t - \sigma_1)) + (b_0 + \beta_2)f(u_2(t - \sigma_2)) + (c_0 + \beta_3)f(u_3(t - \sigma_3)), \quad (14)$$

其中 $\beta = (\beta_1,\beta_2,\beta_3)$ 在原点(0,0,0) 附近充分小. 同样，系统(14) 的泰勒展开式如下：

$$\dot{U}(t) = AU(t) + \sum_{i=1}^{2} B_i U(t - \sigma_i) + \widetilde{B}_3 U(t - \sigma_3) + \frac{1}{2}\widetilde{F}_2(U_t, P) + \cdots. \quad (15)$$

为了获得系统(6) 在原点处的规范型，需要计算系统(7) 的广义特征空间 P 的基 $\Phi(\theta)$ 及其对偶空间 P^* 的基 $\Psi(s)$. 其中 $\Phi(\theta)$ 和 $\Psi(s)$ 的选取详见文献[10] 的引理 3.1. 通过计算，可以得出

$$\Phi(\theta) = \begin{pmatrix} 1 & \theta & \frac{1}{2}\theta^2 + u_{31} \\ 1 & \theta + 1 & \frac{1}{2}\theta^2 + \theta + u_{31} \\ 1 & \theta + 2 & \frac{1}{2}\theta^2 + 2\theta + u_{31} + 1 \\ 1 & \theta + 3 & \frac{1}{2}\theta^2 + 3\theta + u_{31} + 3 \end{pmatrix}, \Psi(0) = \begin{pmatrix} 0 & v_{12} & v_{13} & v_{14} \\ v_{21} & v_{22} & v_{23} & v_{24} \\ v_{31} & v_{32} & v_{33} & v_{34} \end{pmatrix}, \quad (16)$$

其中 $\Phi(\theta)$ 满足 $\dot{\Phi} = \Phi B$，且 $B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$，

$$u_{31} = -\frac{s_3 v_{21} + s_4 v_{31}}{20 s_5 v_{31}}, v_{31} = \frac{6a_0}{a_0\sigma_1^3 + b_0\sigma_2^3 + c_0\sigma_3^3 - 3b_0\sigma_2^2 - 6c_0\sigma_3^2 + 6c_0\sigma_3 + 24},$$

$$v_{32} = \frac{(a_0 + b_0)v_{31}}{a_0}, v_{33} = v_{34} = \frac{v_{31}}{a_0}, v_{21} = -\frac{3a_0 s_2}{2(a_0\sigma_1^3 + b_0\sigma_2^3 + c_0\sigma_3^3 - 3b_0\sigma_2^2 - 6c_0\sigma_3^2 + 6c_0\sigma_3 + 24t)^2},$$

$$v_{22} = \frac{(a_0 + b_0)v_{21} - (a_0 - b_0\sigma_1 + b_0\sigma_2)v_{31}}{a_0}, v_{23} = \frac{(2 + \sigma_1)v_{31} + v_{21}}{a_0},$$

$$v_{24} = \frac{(1 + \sigma_1)v_{31} + v_{21}}{a_0}, v_{12} = -\frac{[a_0 + b_0(\sigma_2 - \sigma_1)]v_{21}}{a_0} + \frac{[b_0(\sigma_1 - \sigma_2)^2 + 2a_0]v_{31}}{2a_0},$$

$$v_{13} = \frac{2(\sigma_1 + 2)\, v_{21} + (\sigma_1^2 + 4\sigma_1 + 2)\, v_{31}}{2a_0}, v_{14} = \frac{2(1 + \sigma_1)\, v_{21} + \sigma_1(\sigma_1 + 2)v_{31}}{2a_0},$$

$$s_2 = 3a_0\sigma_1^4 + 4b_0\sigma_1\sigma_2^3 - b_0\sigma_2^4 + 4c_0\sigma_1\sigma_3^3 - c_0\sigma_3^4 + 4a_0\sigma_1^3 - 12b_0\sigma_1\sigma_2^2 + 8b_0\sigma_2^3 - 24c_0\sigma_1\sigma_3^2 + 12c_0\sigma_3^3 - 12b_0\sigma_2^2 + 24c_0\sigma_1\sigma_3 - 36c_0\sigma_3^2 + 24c_0\sigma_3 + 96\sigma_1 + 120,$$

$$s_3 = 15a_0\sigma_1^4 + 20b_0\sigma_1\sigma_2^3 - 5b_0\sigma_2^4 + 20c_0\sigma_1\sigma_3^3 - 5c_0\sigma_3^4 + 20a_0\sigma_1^3 - 60b_0\sigma_1\sigma_2^2 + 40b_0\sigma_2^3 - 120c_0\sigma_1\sigma_3^2 + 60c_0\sigma_3^3 - 60b_0\sigma_2^2 + 120c_0\sigma_1\sigma_3 - 180c_0\sigma_3^2 + 120c_0\sigma_3 + 480\sigma_1 + 600,$$

$$s_4 = 6a_0\sigma_1^5 + 10b_0\sigma_1^2\sigma_2^3 - 5b_0\sigma_1\sigma_2^4 + b_0\sigma_2^5 + 10c_0\sigma_1^2\sigma_3^3 - 5c_0\sigma_1\sigma_3^4 + c_0\sigma_3^5 + 15a_0\sigma_1^4 - 30b_0\sigma_1^2\sigma_2^2 + 40b_0\sigma_1\sigma_2^3 - 10b_0\sigma_2^4 - 60c_0\sigma_1^2\sigma_3^2 + 60c_0\sigma_1\sigma_3^3 - 15c_0\sigma_3^4 - 60b_0\sigma_1\sigma_2^2 + 20b_0\sigma_2^3 + 60c_0\sigma_1^2\sigma_3 - 180c_0\sigma_1\sigma_3^2 + 60c_0\sigma_3^3 + 120c_0\sigma_1\sigma_3 - 60c_0\sigma_3^2 + 240\sigma_1^2 + 600\sigma_1 + 120,$$

$$s_5 = a_0\sigma_1^3 + 3b_0\sigma_1^2\sigma_2 - 3b_0\sigma_1\sigma_2^2 + b_0\sigma_2^3 + 3c_0\sigma_1^2\sigma_3 - 3c_0\sigma_1\sigma_3^2 + c_0\sigma_3^3 + 3a_0\sigma_1^2 + 3b_0\sigma_1^2 + 6c_0\sigma_1\sigma_3 - 3c_0\sigma_3^2 + 6\sigma_1^2 + 6a_0 + 18\sigma_1 + 6.$$

假设 $\mathrm{Re}\lambda \neq 0$，则系统(15) 能被写成

$$\dot{z} = Bz + \frac{1}{2}g_2^1(z,0,\beta) + \cdots, \tag{17}$$

其中 $g_2^1(z,0,\beta) = (I - P_I, 2^1)f_2{}^1(\widetilde{x},0,\mu)$，$f_2^1(\widetilde{x},0,\mu) = \Psi(0)\widetilde{F}(\Phi z,\beta)$. 因此，由(15)，(16) 和(17) 得到

$$\frac{1}{2}f_2^1(\dot{x},0,\mu) = \psi(0)\left[\frac{1}{2}\widetilde{F}_2(\Phi\hat{x},\mu)\right] = \begin{pmatrix} 0 & v_{12} & v_{13} & v_{14} \\ v_{21} & v_{22} & v_{23} & v_{24} \\ v_{31} & v_{32} & v_{33} & v_{34} \end{pmatrix} \begin{pmatrix} \frac{k}{2}\varphi_2^2(0) \\ \frac{k}{3}\varphi_2^2(0) \\ \frac{k}{4}\varphi_2^2(0) \\ \beta_1\varphi_1(-\sigma_1) + \beta_2\varphi_2(-\sigma_2) + \beta_3\varphi_3(-\sigma_3) + \\ \frac{a_0k}{2}\varphi_1^2(-\sigma_1) + \frac{b_0k}{2}\varphi_2^2(-\sigma_2) + \frac{c_0k}{2}\varphi_3^2(-\sigma_3) \end{pmatrix} =$$

$$\begin{pmatrix} a_{200}x_1^2 + a_{110}x_1x_2 + a_{101}x_1x_3 + v_{14}(\beta_2 + \beta_1 + \beta_3)x_1 + a_{020}x_2^2 + \\ a_{011}x_2x_3 + v_{14}[\beta_2(1-\sigma_2) + \beta_3(2-\sigma_3) - \beta_1\sigma_1]x_2 + a_{002}x_3^2 + \\ v_{14}[\beta_1(\frac{1}{2}\sigma_1^2 + u_{31}) + \beta_2(\frac{1}{2}\sigma_2^2 - \sigma_2 + u_{31}) + p_0\beta_3]x_3 \\ b_{200}x_1^2 + b_{110}x_1x_2 + b_{101}x_1x_3 + v_{24}(\beta_2 + \beta_1 + \beta_3)x_1 + b_{020}x_2^2 + \\ b_{011}x_2x_3 + v_{24}[\beta_2(1-\sigma_2) + \beta_3(2-\sigma_3) - \beta_1\sigma_1]x_2 + b_{002}x_3^2 + \\ v_{24}[\beta_1(\frac{1}{2}\sigma_1^2 + u_{31}) + \beta_2(\frac{1}{2}\sigma_2^2 - \sigma_2 + u_{31}) + p_0\beta_3]x_3 \\ c_{200}x_1^2 + c_{110}x_1x_2 + c_{101}x_1x_3 + v_{34}(\beta_2 + \beta_1 + \beta_3)x_1 + c_{020}x_2^2 + \\ c_{011}x_2x_3 + v_{34}[\beta_2(1-\sigma_2) + \beta_3(2-\sigma_3) - \beta_1\sigma_1]x_2 + c_{002}x_3^2 + \\ v_{34}[\beta_1(\frac{1}{2}\sigma_1^2 + u_{31}) + \beta_2(\frac{1}{2}\sigma_2^2 - \sigma_2 + u_{31}) + p_0\beta_3]x_3 \end{pmatrix}, \tag{18}$$

其中

$$a_{200} = \frac{k}{2}(v_{12} + v_{13} + v_{14}),\ a_{110} = k(2v_{12} + 3v_{13} + 4v_{14}),$$

$$a_{101} = k[(u_{31} + 1)v_{12} + (u_{31} + 3)v_{13} + v_{14}(u_{31} + 6)],$$

$$a_{020} = \frac{k}{2}[((\sigma_2^2 - 2\sigma_2 + 1)b_0 + (\sigma_3^2 - 4\sigma_3 + 4)c_0 + a_0\sigma_1^2)v_{14} + 4v_{12} + 9v_{13}],$$

$$a_{011} = \frac{k}{2}[4(u_{31} + 1)v_{12} + 6(u_{31} + 3)v_{13} - (\sigma_1(\sigma_1^2 + 2u_{31})a_0 + (\sigma_2 - 1)(\sigma_2^2 + 2u_{31} - 2\sigma_2)b_0 + (\sigma_3 - 2)(\sigma_3^2 + 2u_{31} - 4\sigma_3 + 2)c_0)v_{14}],$$

$$a_{002} = \frac{k}{8}[4(u_{31} + 1)^2v_{12} + 4(u_{31} + 3)^2v_{13} + ((\sigma_1^2 + 2u_{31})^2a_0 + (\sigma_2^2 + 2u_{31} - 2\sigma_2)^2b_0 + (\sigma_3^2 + 2u_{31} - 4\sigma_3 + 2)^2c_0)v_{14}],$$

$$b_{200}=\frac{k}{2}(v_{21}+v_{22}+v_{23}+v_{24}),b_{110}=k(v_{21}+2v_{22}+3v_{23}+4v_{24}),$$

$$b_{101}=k[u_{31}v_{21}+(u_{31}+1)\ v_{22}+(u_{31}+3)v_{23}+(u_{31}+6t)v_{24}],$$

$$b_{020}=\frac{k}{2}[((\sigma_2^2-2\sigma_2+1)\ b_0+(\sigma_3^2-4\sigma_3+4)\ c_0+a_0\sigma_1^2)v_{24}+9v_{23}+4v_{22}+v_{21}],$$

$$b_{011}=\frac{k}{2}[2u_{31}v_{21}+4(u_{31}+1)\ v_{22}+6(u_{31}+3)\ v_{23}-(\sigma_1(\sigma_1^2+2u_{31})a_0+$$

$$(\sigma_2-1)(\sigma_2^2+2u_{31}-2\sigma_2)b_0+(\sigma_3-2)(\sigma_3^2+2u_{31}-4\sigma_3+2)\ c_0)\ v_{24}],$$

$$b_{002}=\frac{k}{8}[4(1+u_{31})^2v_{22}+4(3+u_{31})^2v_{23}+((\sigma_1^2+2u_{31})^2a_0+(\sigma_2^2+$$

$$2u_{31}-2\sigma_2)^2b_0+c_0(\sigma_3^2+2u_{31}-4\sigma_3+2)^2)v_{24}+4u_{31}^2v_{21}],$$

$$c_{200}=\frac{k}{2}(v_{31}+v_{32}+v_{33}+v_{34}),c_{110}=k(v_{31}+2v_{32}+3v_{33}+4v_{34}),$$

$$c_{101}=k[u_{31}v_{31}+(1+u_{31})\ v_{32}+(3+u_{31})\ v_{33}+(u_{31}+6)\ v_{34}],$$

$$c_{020}=\frac{k}{2}[((\sigma_2^2-2\sigma_2+1)\ b_0+(\sigma_3^2-4\sigma_3+4)\ c_0+a_0\sigma_1^2)v_{34}+9v_{33}+4v_{32}+v_{31}],$$

$$c_{002}=\frac{k}{8}[4(1+u_{31})^2v_{32}+4(3+u_{31})^2v_{33}+((\sigma_1^2+2u_{31})^2a_0+(\sigma_2^2+2u_{31}-2\sigma_2)^2b_0+$$

$$c_0(\sigma_3^2+2u_{31}-4\sigma_3+2^2)v_{34}+4u_{31}^2v_{31}],p_0=\frac{1}{2}\sigma_3^2-2\sigma_3+u_{31}+1.$$

由系统(17)和(18),可以得到

$$\dot{x}_1=x_2,\dot{x}_2=x_3,\dot{x}_3=k_1x_1+k_2x_2+k_3x_3+h_1x_1^2+h_2x_2^2+h_3x_1x_2+h_4x_1x_3+\cdots. \tag{19}$$

其中

$$k_1=v_{34}(\beta_1+\beta_2+\beta_3),k_2=(v_{24}-v_{34}\sigma_1)\beta_1+(v_{34}(1-\sigma_2)+v_{24})\beta_2+(v_{34}(2-\sigma_3)+v_{24})\beta_3,$$

$$k_3=[v_{34}(\frac{1}{2}\sigma_1^2+u_{31})+v_{14}-v_{24}\sigma_1]\beta_1+[(\frac{1}{2}\sigma_2^2-\sigma_2+u_{31})v_{34}+(1-\sigma_2)\ v_{24}+v_{14}]\beta_2+$$

$$[v_{34}(\frac{1}{2}\sigma_3^2-2\sigma_3+u_{31}+1)+v_{14}+(2-\sigma_3)\ v_{24}]\beta_3,h_1=c_{200},h_2=2a_{200}+b_{110+}c_{020},$$

$$h_3=2b_{200}+c_{110},h_4=a_{101}+2a_{200}+b_{110}.$$

容易计算出

$$\left|\frac{\partial(k_1,k_2,k_3)}{\partial(\beta_1,\beta_2,\beta_3)}\right|_{\beta=0}=\frac{108[(\sigma_2-\sigma_3+1)\sigma_1^2+g_1\sigma_1+g_0]^4}{(d_1\sigma_1^3+d_2\sigma_1^2+d_3\sigma_1+d_4)^3}, \tag{20}$$

其中

$$g_0=(\sigma_3-2)\sigma_2^2+(-\sigma_3^2+2\sigma_3+2)\sigma_2+\sigma_3^2-4\sigma_3+2,\ g_1=-\sigma_2^2+\sigma_3^2+2\sigma_2-4\sigma_3+2,$$

$$d_1=(\sigma_3+2)\sigma_2^2+(-\sigma_3^2+2\sigma_3+6)\ \sigma_2-3\sigma_3^2+6,$$

$$d_2=-(\sigma_3+2)\ \sigma_2^3+3(\sigma_3+2)\sigma_2^2+(\sigma_3^3-6\ \sigma_3^2+6\sigma_3+24)\ \sigma_2+3\sigma_3{}^3-18\sigma_3^2-6\sigma_3+24,$$

$$d_3=(\sigma_3^2-4\sigma_3-10)\ \sigma_2^3+(-\sigma_3^3+3\sigma_3^2+6\sigma_3+6)\sigma_2^2+2(\sigma_3^3-6\sigma_3^2+6\sigma_3+24)\ \sigma_2+$$

$$12\sigma_3^3-48\sigma_3^2-24\sigma_3+48,$$

$$d_4=4(\sigma_3^2-\sigma_3-4)\sigma_2^3+4\sigma_3(-\sigma_3^2+3\sigma_3+3)\sigma_2^2+4(-\sigma_3^3+6\sigma_3+12)\ \sigma_2+12\sigma_3^3-48\sigma_3^2-24\sigma_3+48.$$

令 $G(\sigma_1)=(\sigma_2-\sigma_3+1)\sigma_1^2+g_1\sigma_1+g_0$,则 $G(\sigma_1)=0$ 的判别式为

$$\Delta=g_1^2-4(\sigma_2-\sigma_3+1)g_0=[(\sigma_2-\sigma_3)^2-2][(\sigma_2-\sigma_3+4)(\sigma_2-\sigma_3)+2],$$

因为 $\sigma_2>\sigma_3>0$,可以推出 $\sigma_2-\sigma_3+1>0,(\sigma_2-\sigma_3+4)(\sigma_2-\sigma_3)>0$. 其次,假设 $0<\sigma_2-\sigma_3<\sqrt{2}$,得到 $(\sigma_2-\sigma_3)^2-2<0$. 因此,$\Delta<0$,由此知 G 没有实根,即 $\left|\frac{\partial(k_1,k_2,k_3)}{\partial(\beta_1,\beta_2,\beta_3)}\right|_{\beta=0}\neq 0$.

因此,可以得到下面的定理.

定理 2 如果 $k=f''(0)\neq 0$,且引理 1 成立,则系统(19) 在中心流形上等价于系统(14).

根据文献[11],系统(19) 在原点处的分支分析如下:

1) 当 $T = \{(\beta_1,\beta_2,\beta_3):k_1 = 0\}$ 时，系统(19) 经历一个超临界分支.

2) 当 $H_1 = \{(\beta_1,\beta_2,\beta_3):k_3 = -\frac{k_1}{k_2},k_2 < 0\}$ 时，系统(19) 经历一个 Hopf 分支.

3) 当 $H_2 = \left\{(\beta_1,\beta_2,\beta_3):k_3 = \left(\frac{h_4}{h_1} - \frac{h_1}{h_3k_1 - h_1k_2}\right)k_1,\frac{h_1}{h_3k_1 - h_1k_2} > 0\right\}$ 时，系统(19) 在平衡点$(-\frac{g_1}{h_1},0,0)$ 处经历一个 Hopf 分支.

(4) 当 $BT = \{(\beta_1,\beta_2,\beta_3):\ k_1 = 0,\ k_2 = 0\}$ 时，系统(19) 经历一个 B-T 分支.

(5) 当 $H_3 = \{(\beta_1,\beta_2,\beta_3):\ k_1 = 0,\ k_3 = 0,\ k_2 < 0\}$ 时，系统(19) 经历一个 zero-Hopf 分支.

如果 $k = f''(0) = 0$，则 $\gamma_1 = \gamma_2 = 0,h_1 = h_2 = 0$，即系统(13)，(19) 是退化的. 此时，为了分析系统(2)在 B-T 奇点和 triplezero 奇点附近的动力学性质，需要进一步计算相应的三阶规范型. 该问题将在以后的文章中做进一步探讨.

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Bifurcation Analysis in a Recurrent Neural Network Model with Delays

LIU Xia, JIAO Jianfeng

(Henan Engineering Laboratory for Big Data Statistical Analysis and Optimal Control; College of Mathematics and Information Science, Henan Normal University, Xinxiang 453007, China)

Abstract: In this paper, we pay our main attention to study the bifurcation analysis of a four-node recurrent neural network model with six discrete delays. By using center manifold reduction and normal form method of delay differential equations, the norm forms of the Bogdanov-Takens (B-T) and triple zero bifurcations at origin are obtained. Finally, we gave some main bifurcation phenomenons.

Keywords: neural network model; B-T bifurcation; triple zero bifurcation; normal form